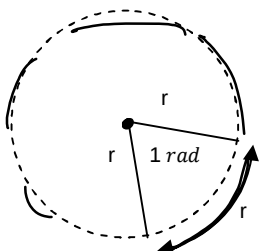


Unit 4: Trigonometry & The Unit Circle

4.1 Angles & Angle Measure

Alternative to measuring angles in degrees $\Rightarrow$ radian measure

Consider a circle with radius r .



Radian: One radian (1 rad or 1) is the measure of the central angle subtended in a circle by an arc equal in length to the radius.

How many radians on one full revolution? (360°)

ie. How many arcs of length r will fit around the circle?

$$\text{Circumference } C = 2\pi r$$

$$\therefore \frac{2\pi r}{r} \leftarrow \begin{array}{l} \text{total distance around} \\ \text{arc length of 1 radian} \end{array}$$

$$360^\circ = 2\pi \text{ rads}$$

$$180^\circ = \pi \text{ rads}$$

Ex. Draw each angle in standard position. Change angles in degrees to radians and angles in radians to degrees. Express answers in both exact and approximate measures to the nearest hundredth.

a) 30°



$$\frac{\pi}{180} = \frac{\theta}{30}$$

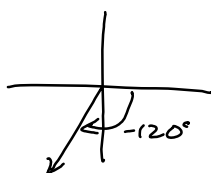
$$\theta = \frac{30\pi}{180}$$

$$\theta = \pi/6 \leftarrow \text{exact radian form}$$

$$\approx 0.52$$

b) -120°

negative angles
clockwise rotation



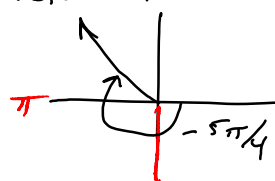
$$\frac{\pi}{180} = \frac{\theta}{-120}$$

$$\theta = \frac{-120\pi}{180}$$

$$\theta = -\frac{2\pi}{3} \approx -2.09$$

c) $-\frac{5\pi}{4}$

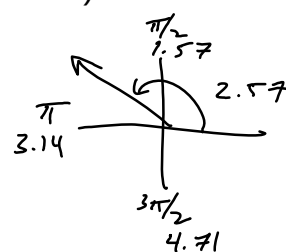
$5 \times \frac{\pi}{4}$



$$\theta = -5(180^\circ)$$

$$\theta = -225^\circ$$

d) 2.57

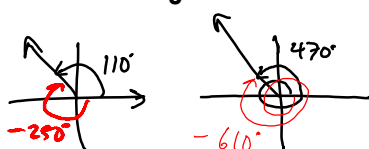


$$\frac{\pi}{180} = \frac{2.57}{\theta}$$

$$\theta = 180(2.57)$$

$$\theta = \frac{\pi}{180} \cdot 147.25^\circ$$

Coterminal Angles:



Angles in standard position with the same terminal arm

Ex. Determine the quadrant in which the terminal arm of each angle lies and find one positive and one negative coterminal angle.

a) 40° Q I

$$40^\circ - 360^\circ = -320^\circ$$

$$40^\circ + 360^\circ = 400^\circ$$

b) $-430^\circ + 360^\circ$ Q IV

$$-70^\circ$$

$$-70^\circ + 360^\circ = 290^\circ$$

c) $\frac{8\pi}{3}$ Q II
 $-2\pi \frac{6\pi}{3}$

$$\frac{2\pi}{3}$$

$$2\pi/3 - 6\pi/3 = -4\pi/3$$

$$\uparrow$$

$$2\pi$$

Coterminal Angles in General Form:

Add or Subtract full rotations

∞ possibilities
for each terminal
arm

Coterminals
of a given: $\theta \pm 360^\circ n$ OR $\theta \pm 2\pi n$
 $n \in \mathbb{N}$

Ex.

a) Express angles coterminal with 110° in general form. Find all coterminal angles such that

$$-720^\circ \leq \theta \leq 720^\circ$$

$$\boxed{110^\circ \pm 360^\circ n, n \in \mathbb{N}}$$

$$-720^\circ \leftarrow -360^\circ \quad 110^\circ \quad +360^\circ \rightarrow 720^\circ$$

$$\boxed{-610^\circ, -250^\circ, 470^\circ}$$

b) Express angles coterminal with $\frac{8\pi}{3}$ in general form. Find all coterminal angles such that

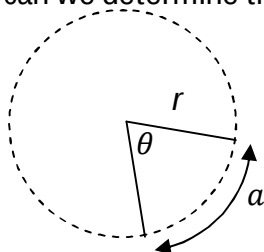
$$-4\pi \leq \theta \leq 4\pi$$

$$\boxed{\frac{8\pi}{3} \pm 2\pi n, n \in \mathbb{N}}$$

$$-4\pi \leftarrow -2\pi \quad \frac{8\pi}{3} \quad +2\pi \rightarrow 4\pi$$

$$\boxed{-\frac{10\pi}{3}, -\frac{4\pi}{3}, \frac{2\pi}{3}}$$

How can we determine the length of an arc in a circle?



Use a proportion comparing the sector angle θ to a full revolution and arc length to the full circumference (c)

$$\frac{a}{c} = \frac{\theta}{1 \text{ revolution}}$$

Degrees

$$\frac{a}{2\pi r} = \frac{\theta}{360^\circ}$$

$$a = \frac{2\pi r \theta}{360}$$

$$a = \frac{\pi r \theta}{180}$$

Rads

$$\frac{a}{2\pi r} = \frac{\theta}{2\pi}$$

$$a = \frac{2\pi r \theta}{2\pi}$$

$$\boxed{a = r\theta} \quad \text{RADS!!}$$

Ex. Provide the missing information, to the nearest tenth, in each case below.

a) $r = 8.7 \text{ cm}$, $\theta = 75^\circ$,
 $a = ?$

$$a = r\theta$$

$$= 8.7 \left(\frac{75\pi}{180} \right)$$

$$= \boxed{11.4 \text{ cm}}$$

$$\frac{\pi}{180} = \frac{\theta}{75}$$

$$\theta = \frac{75\pi}{180} = \frac{5\pi}{12}$$

b) $a = 4.7 \text{ mm}$, $\theta = 1.8$,
 $r = ?$

$$a = r\theta$$

$$r = \frac{a}{\theta}$$

$$= \frac{4.7}{1.8}$$

$$r = \boxed{2.6 \text{ cm}}$$

c) $r = 5 \text{ m}$, $a = 13 \text{ m}$,
 $\theta = ?$

$$a = r\theta$$

$$\theta = \frac{a}{r}$$

$$= \frac{13}{5}$$

$$\theta = \boxed{2.6}$$