Rationalizing the Denominator

Expressions are not considered to be <u>Completely simplified</u> if they contain a radical in the denominator. The act of rationalizing (<u>removing the radical</u>) from the denominator is called <u>Rationalizing the Denominator</u>. There are 2 cases:

A) For an expression with a <u>monomial</u> square root denominator, multiply both the <u>numerator & denominator</u> by the radical term.

<u>Example 1:</u> Rationalize the denominator & simplify

a) $\dfrac{5}{2\sqrt{3}}\left(\dfrac{\sqrt{3}}{\sqrt{3}}\right)$

$\dfrac{5\sqrt{3}}{2\sqrt{9}} \rightarrow \dfrac{5\sqrt{3}}{6}$

b) $\dfrac{7+3\sqrt{3}}{6\sqrt{6}}\left(\dfrac{\sqrt{6}}{\sqrt{6}}\right)$

$\dfrac{7\sqrt{6}+3\sqrt{18}}{6\sqrt{36}} \rightarrow \dfrac{7\sqrt{6}+9\sqrt{2}}{36}$

c) $\dfrac{a\sqrt{b}}{c\sqrt{d}}\left(\dfrac{\sqrt{d}}{\sqrt{d}}\right)$

$\dfrac{a\sqrt{bd}}{cd}$

d) $\dfrac{5+\sqrt[3]{10x^2}}{4\sqrt[3]{4x}}\left(\dfrac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}}\right)$

$\dfrac{5\cdot\sqrt[3]{2x^2}+\sqrt[3]{20x^4}}{4\sqrt[3]{8x^3}} \rightarrow \dfrac{5\cdot\sqrt[3]{2x^2}+x\sqrt[3]{20x}}{8x}$

<u>Conjugates:</u> two binomial factors whose product is a <u>difference of squares</u>.

e.g. (x + 5)(x – 5) $\rightarrow x^2-25$ (a + b)(a – b) $\rightarrow a^2-b^2$

<u>Example 2:</u> simplify the products of conjugates:

a) $\left(5+\sqrt{3}\right)\left(5-\sqrt{3}\right)$

$25-9 \rightarrow 16$

b) $\left(2\sqrt{6}+\sqrt{20}\right)\left(2\sqrt{6}-\sqrt{20}\right)$

$4\sqrt{36}-20$

$24-20 \rightarrow 4$

c) $\left(2x+\sqrt{x}\right)\left(2x-\sqrt{x}\right)$

$4x^2-x$ or $x(4x-1)$ ← No Radicals!

B) For a binomial denominator that contains a square root, multiply both the numerator & denominator by a conjugate of the denominator.
(Why? product will be Rational)

Example 2: Rationalize the denominator & simplify

a) $\dfrac{11}{\sqrt{5}+7}\left(\dfrac{\sqrt{5}-7}{\sqrt{5}-7}\right)$

$\dfrac{11\sqrt{5}-77}{5-49}$

$\dfrac{11\sqrt{5}-77}{-44}$

b) $\dfrac{6}{\sqrt{4x}+1}\left(\dfrac{\sqrt{4x}-1}{\sqrt{4x}-1}\right)$

$\dfrac{6\sqrt{4x}-6}{4x-1}$ done

c) $\dfrac{4+4\sqrt{4}}{2+2\sqrt{2}}$

$=\dfrac{4+8}{2+2\sqrt{2}}$

$=\dfrac{12}{2+2\sqrt{2}}$

$=\dfrac{6}{1+\sqrt{2}}\left(\dfrac{1-\sqrt{2}}{1-\sqrt{2}}\right)$

$=\dfrac{6-6\sqrt{2}}{1-2}$

$=\dfrac{6-6\sqrt{2}}{-1} \rightarrow 6\sqrt{2}-6$

d) $\dfrac{5+2\sqrt{15}}{5-\sqrt{15}}\left(\dfrac{5+\sqrt{15}}{5+\sqrt{15}}\right)$

$\dfrac{25+5\sqrt{15}+10\sqrt{15}+2\sqrt{225}}{25-15}$

$\dfrac{25+15\sqrt{15}+30}{10}$

$\dfrac{55+15\sqrt{15}}{10}$ simplify

$\dfrac{11+3\sqrt{15}}{2}$

Assignment: p. 291 # 8 – 11, 14, 20,