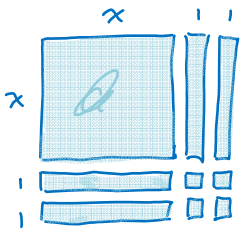
PreCalculus 11 Name_____
Date_____

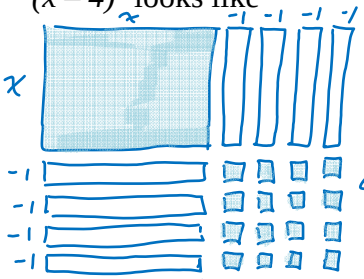
3.3 Completing the Square

Remember Algebra Tiles? We can make a perfect square using x^2's, x's and 1's:

e.g. $(x + 2)^2$ looks like x^2+4x+4



$(x - 4)^2$ looks like $x^2-8x+16$



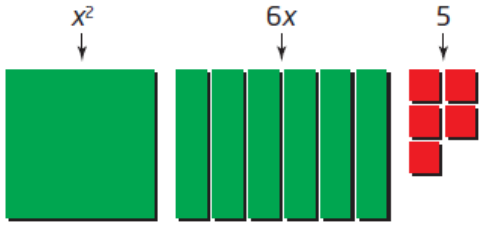
(−)(−) gives (+)

If we don't have all the necessary pieces, we may have to add on zero pairs

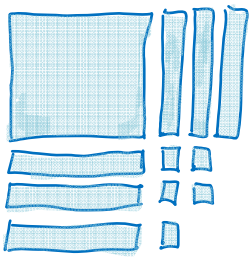
in order to complete the square.

Example 1: p. 184

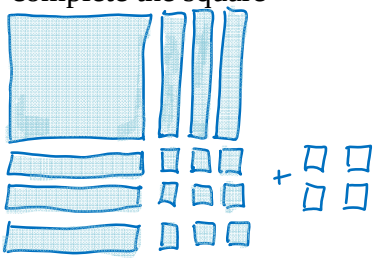
a) complete the square for $x^2 + 6x + 5$ and rewrite as



reorganize as:



add on +4 & −4 to complete the square



final result: $(x+3)^2-4$

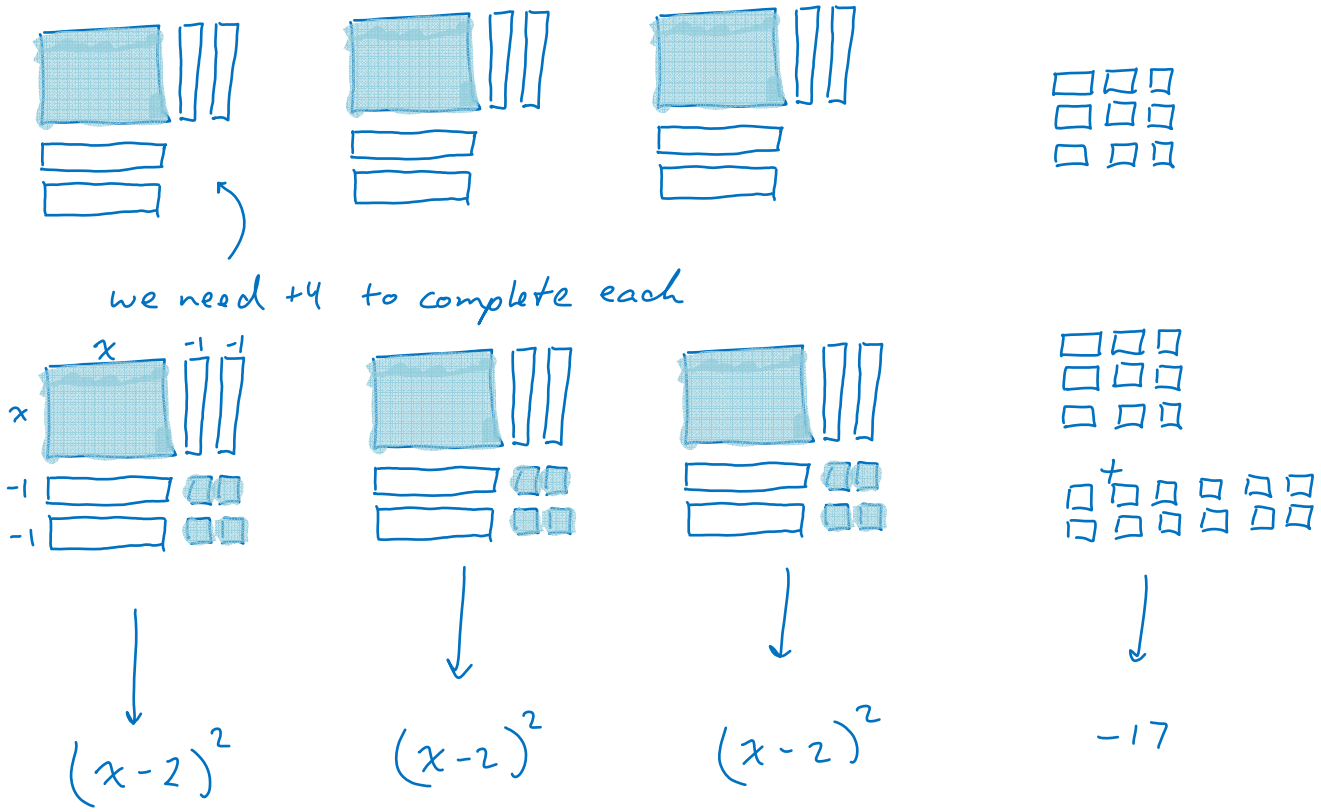
Notice that this is the same as what we started with:

i.e. $(x+3)^2-4$

$(x+3)(x+3)-4 \longrightarrow x^2+6x+9-4 \longrightarrow x^2+6x+5$

b) complete the square and rewrite $3x^2 - 12x - 9$

reorganize into 3 squares:



$\rightarrow \quad 3(x-2)^2 - 17$

We can also accomplish this through algebra. To start with, consider a perfect square:

e.g. $(x + 6)^2 =$ $x^2 + 12x + 36$

$(x - 13)^2 =$ $x^2 - 26x + 169$

$(3x + 7)^2 =$ $9x^2 + 42x + 49$

Shortcut: Square 1st + 2(1st)(2nd) + Square 2nd

What if the last number is missing?

e.g. $x^2 + 10x + \underline{25}$ → $(x+5)^2$

$x^2 - 26x + \underline{169}$ → $(x-13)^2$

What about $2x^2 + 24x + \underline{72}$ → $2(x+6)^2$

$2(x^2+12x+36)$

To complete the square, factor out the a from first 2 terms, then add on $\left(\frac{2^{nd}}{2}\right)^2$ → and subtract off too!.

Example 2: p. 184 - rewrite in vertex form by completing the square:

a) $f(x) = x^2 + 6x + 5$

$= x^2 + 6x + 9 - 9 + 5$

$= (x+3)^2 - 9 + 5$

$f(x) = (x+3)^2 - 4$

b) $f(x) = 3x^2 - 12x - 9$

$= 3(x^2 - 4x + 4 - 4) - 9$

$= 3(x-2)^2 - 12 - 9$

$f(x) = 3(x-2)^2 - 21$

c) $f(x) = -5x^2 - 70x$

$f(x) = -5(x^2 + 14x)$

$= -5(x^2 + 14x + 49 - 49)$

$= -5(x+7)^2 + 245$

Example 3: p. 187

Convert $y = 4x^2 - 28x - 23$ to vertex form and verify the two forms are equivalent.

$y = 4(x^2 - 7x\ ) - 23$

hmmmm..... divide by 2: $\frac{7}{2}$

then square: $\frac{49}{4}$

$= 4\left(x^2 - 7x + \frac{49}{4} - \frac{49}{4}\right) - 23$

$= 4\left(x - \frac{7}{2}x\right)^2 - 49 - 23$

$y = 4\left(x - \frac{7}{2}\right)^2 - 72$

expand it out: $y = 4\left(x^2 - 7x + \frac{49}{4}\right)^2 - 72$

$= 4x^2 - 28x + 49 - 72$

$= 4x^2 - 28x - 23$ yay (:)

Assignment: p. 192 # 1-3