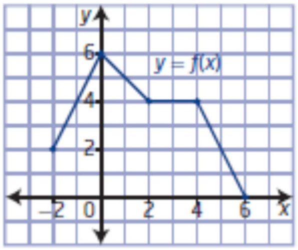
PreCalculus 12

Name_______________
Date_______________

<u>*1.2 Reflections & Stretches*</u>

Starting with the graph of the function $y = f(x)$, and determine 5 important points:

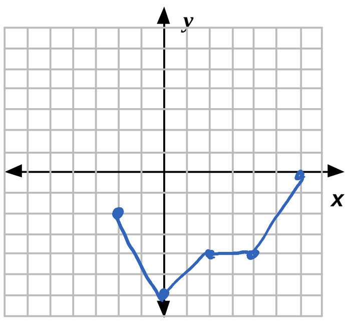


x	y
-2	2
0	6
2	4
4	4
6	0

(x, y)

$(x, -y)$

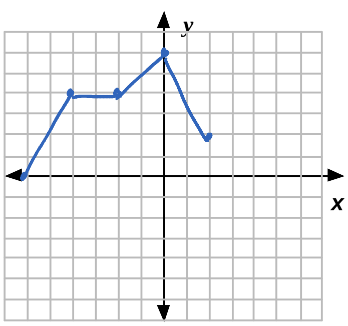
Draw the mirror image of $f(x)$ over the x-axis, and determine the new 5 points:



x	y
-2	-2
0	-6
2	-4
4	-4
6	0

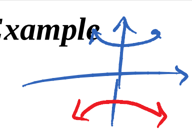
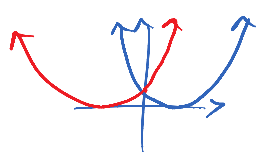
Draw the mirror image of $f(x)$ over the y-axis, and determine the new 5 points: (x, y)

$(-x, y)$



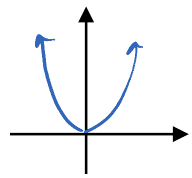
x	y
2	2
0	6
-2	4
-4	4
-6	0

A ***reflection*** is a transformation where each point of the original graph has an __image point__ resulting from a reflection in a line. This may result in a __change in orientation__ of the graph while preserving its __shape__.

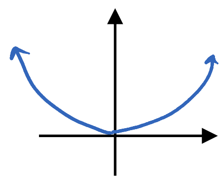
Function	*Transformation*	*Mapping*	*Example*
$y = -f(x)$	Vertical Reflection over x-axis	$(x, y) \to (x, -y)$	
$y = f(-x)$	Horizontal Reflection over y-axis	$(x, y) \to (-x, y)$	

An ***invariant point*** is a point on a graph that remains unchanged, and is a point on the original function that intersects the line of reflection.

We can also ***stretch*** functions vertically and horizontally by introducing a scale factor to one or both of the variables. Starting with a regular parabola, $y = x^2$



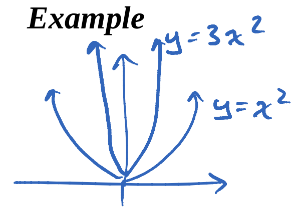
if we change the y to a $2y$, we get $2y = x^2$, or more commonly, $y = \frac{1}{2}x^2$, and a new graph:



"half as tall"

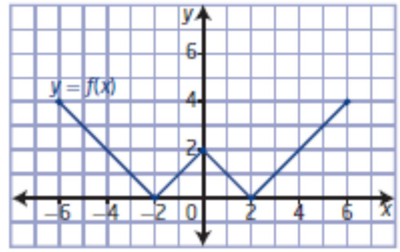
The result is half as tall as the original, so consider:

$3y = f(x)$, or	$y = \frac{1}{3}f(x)$	makes the graph $\frac{1}{3}$	as tall
$\frac{y}{2} = f(x)$	$y = 2f(x)$	makes the graph $2\times$	as tall
$y/7 = f(x)$	$y = 7f(x)$	makes the graph $7\times$	as tall

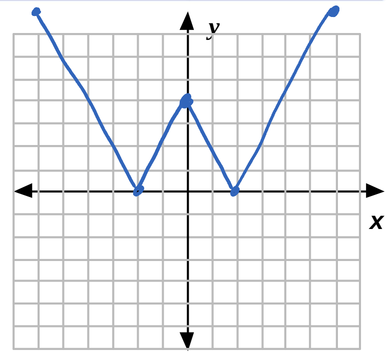
Function	***Transformation***	***Mapping***	***Example***
$y = af(x)$	Vertical stretch by a factor of a	$(x, y) \rightarrow (x, ay)$	

In this case, the invariant points are where the height equals zero, i.e. the x-intercepts of the function.

Example 1: p. 21 – given the graph of $y = f(x)$:



a) sketch and transform the graph to $g(x) = 2f(x)$



b) describe the transformation

Vertical Stretch by 2

c) give any invariant points

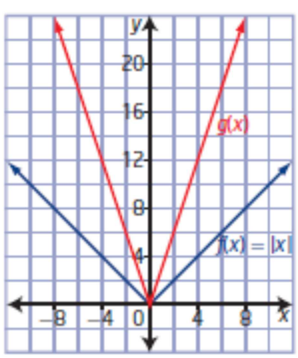
$(\pm 2, 0)$

d) give the domain & range for both $f(x)$ and $g(x)$

Function:	***f(x)*** Domain	***g(x)*** Range
Set Notation	$\{x \mid -6 \le x \le 6\}$	$\{y \mid 0 \le y \le 8\}$
Interval Notation	$[-6, 6]$	$[0, 8]$

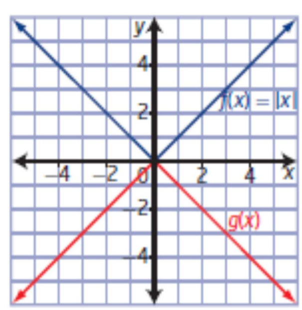
Example 2: p. 25 – Give the equations of the transformed function, $g(x)$:

a)



$g(x) = 3|x|$

b)



$g(x) = -|x|$

Assignment Part 1: p.28 # 2-5, 10

1.2 Function Reflections – Day 2

Now, instead of simply stretching functions vertically, we will stretch them horizontally by introducing a scale factor to the x.

Start with our favorite function, $y = x^2$:

x	y
-2	4
-1	1
0	0
1	1
2	4

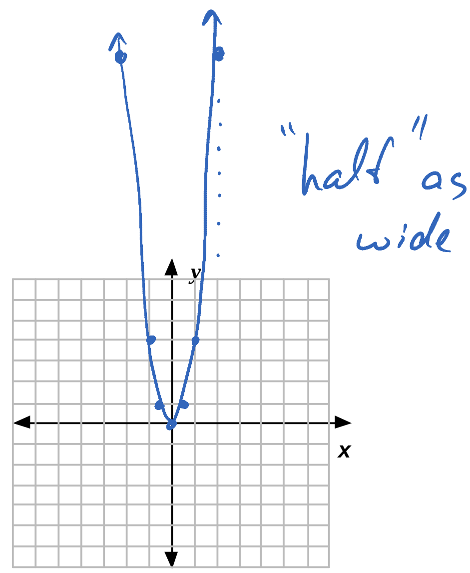
if we replace the x with $2x$,

$y = (2x)^2$

or

$y = 4x^2$

x	y
-2	16
-1	4
0	0
1	4
2	16



The graph has been horizontally compressed and the mapping would be $(x, y) \to (\frac{1}{2}x, y)$.

Function	Transformation	Mapping	Example
$y = f(bx)$	horiz. stretch by $1/b$	$(x, y) \to (\frac{x}{b}, y)$	
$y = f(\frac{x}{b})$	horiz. stretch by b	$(x, y) \to (bx, y)$	

In this case, the invariant points are where the width $= 0$, i.e. the y-intercepts of the function.

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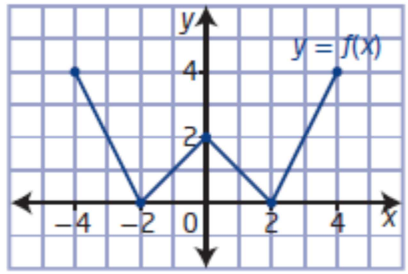
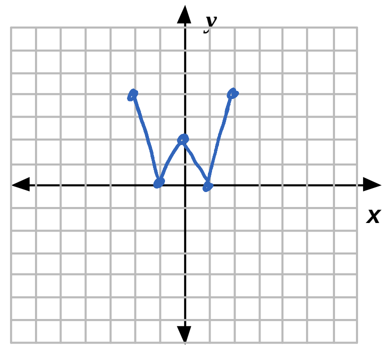
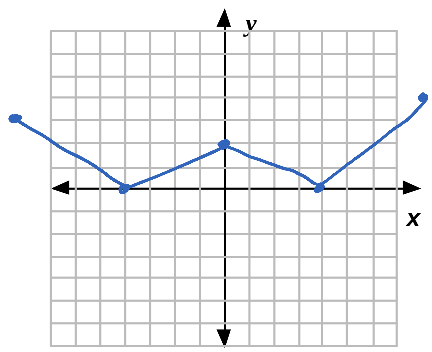
Example 3: p. 23 – given the graph of $y = f(x)$:



a) graph $g(x) = f(2x)$



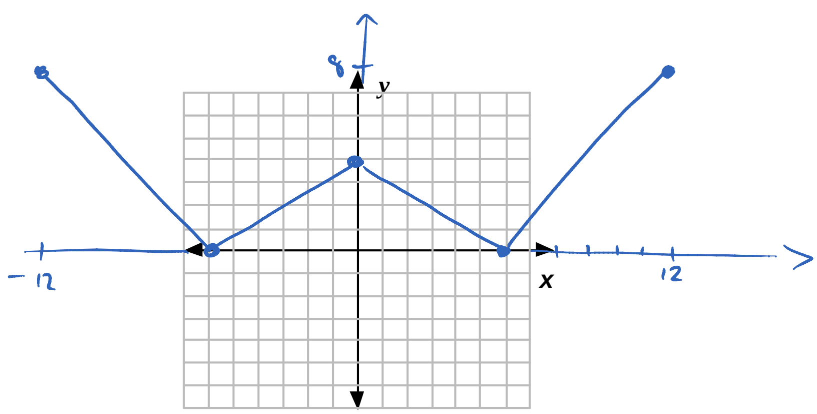
b) graph $g(x) = f\left(\dfrac{x}{2}\right)$



c) graph $g(x) = 2f\left(\dfrac{x}{3}\right)$

2x as tall

3x as wide



Assignment Part 2: p.29 # 6, 7, 9, 14, C2, C3