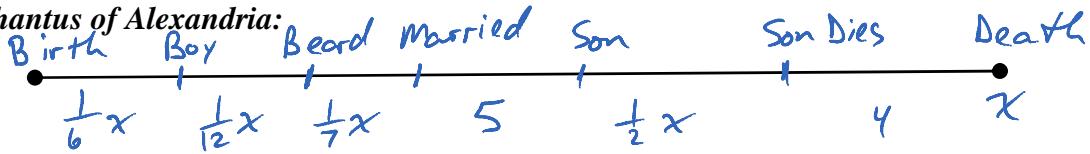


6.4 Solving Rational Equations

Diophantus of Alexandria:



$$\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4 = x$$

Multiply by LCD of 6, 12, 7 $\rightarrow 84$

$$\frac{84}{6}x + \frac{84}{12}x + \frac{84}{7}x + 420 + \frac{84}{2}x + 336 = 84x$$

$$14x + 7x + 12x + 420 + 42x + 336 = 84x$$

$$756 = 9x$$

$$\underline{\underline{84 = x}}$$

He lived to be 84 yrs old.

To solve problems, we end up developing & solving equations, where the thing we are looking for is usually defined to be x.

Expression:

$$\frac{4}{3} + \frac{2}{5}$$

\downarrow
answer is fraction

Equation:

$$\left(\frac{4}{3} + x = \frac{2}{5} \right) \text{ multiply by 15 to get rid of fraction!}$$

The biggest difference in dealing with rational equations, is that we can now multiply both sides by the LCD in order to cancel out the denominators. We still need to factor to find the LCD, and we will also have give non-permissible values.

Example 1: p.342

$$\frac{2}{x^2-4} + \frac{10}{6x+12} = \frac{1}{x-2} \quad \rightarrow x \neq \pm 2$$

LCD :

$$6(x+2)(x-2)$$

$$6(x+2)(x-2) \left(\frac{2}{(x+2)(x-2)} + \frac{10}{6(x+2)} = \frac{1}{x-2} \right)$$

$$12 + 10(x-2) = 6(x+2)$$

$$12 + 10x - 20 = 6x + 12$$

$$10x - 8 = 6x + 12$$

$$4x = 20$$

$$\underline{\underline{x = 5}}$$

Example 2: Solve for k:

$$\frac{4k-1}{k+2} - \frac{k+1}{k-2} = \frac{k^2-4k+24}{k^2-4} \quad \text{LCD } (k+2)(k-2)$$

$$\left(\frac{4k-1}{k+2} - \frac{k+1}{k-2} = \frac{(k+6)(k-4)}{(k+2)(k-2)} \right) \text{LCD } (k+2)(k-2)$$

$$(4k-1)(k-2) - (k+1)(k+2) = (k+6)(k-4)$$

$$4k^2 - 8k - k + 2 - (k^2 + 2k + k + 2) = k^2 - 4k + 6k - 24$$

$$4k^2 - 9k + 2 - k^2 - 3k - 2 = k^2 + 2k - 24$$

$$3k^2 - 12k = k^2 + 2k - 24$$

$$k^2 - 14k + 24 = 0$$

$$(k-2)(k-12) = 0$$

$$\text{K=2 or K=12}$$

No!
Non-perm!

only 1 answer
K=12

Example 3: p.346 - A championship dog race runs non-stop from The Pas to Flin Flon and back.

The total distance is 140 km, and the winner ended up decreasing her speed by 6 km/h on the return trip. If the total time was 8.5 hrs, what was the winner's average speed for the first leg to Flin Flon?

	Dist	Rate	Time
The Pas \rightarrow Flin Flon	70	x	$\frac{70}{x}$
Flin Flon \rightarrow The Pas	70	$x-6$	$\frac{70}{x-6}$
			$\frac{70}{x} + \frac{70}{x-6}$
			<u>8.5</u>

Need to use



$$\text{Equation: } \frac{70}{x} + \frac{70}{x-6} = 8.5$$

$$\frac{70(x)(x-6)}{x} + \frac{70(x)(x-6)}{x-6} = 8.5(x)(x-6)$$

$$70(x-6) + 70(x) = 8.5(x^2 - 6x)$$

$$70x - 420 + 70x = 8.5x^2 - 51x$$

$$140x - 420 = 8.5x^2 - 51x$$

$$0 = 8.5x^2 - 191x + 420$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{191 \pm \sqrt{36481 - 4(8.5)(420)}}{17}$$

$$= \frac{191 \pm \sqrt{22201}}{17}$$

9.4 km/h
2.5 km/h

~~Too small~~

Her avg. speed was 9.4 km/h.