

### 5.3 Solving Radical Equations

A **Radical Equation** is an equation involving radicals with variables in the radicand.

Because of this, we will have give restrictions on variables that need to be stated before we start to solve.

To solve a radical equation, we will isolate the radical, then square both sides, and finally solve for the variable.

**Example 1:** p.296 – State restrictions, then solve for  $x$ :

$$\begin{array}{l}
 5 + \sqrt{2x-1} = 12 \\
 \swarrow \quad \searrow \\
 \sqrt{2x-1} = 7 \qquad 2x-1 \geq 0 \\
 \qquad \qquad \qquad 2x \geq 1 \\
 \qquad \qquad \qquad x \geq \frac{1}{2} \\
 (\sqrt{2x-1})^2 = 7^2 \\
 2x-1 = 49 \\
 2x = 50 \\
 \underline{x = 25}
 \end{array}$$

We should always check our solutions, as squaring both sides may give us Extraneous Roots.

$$\begin{array}{l}
 \underline{\text{Check:}} \quad 5 + \sqrt{2(25)-1} = 12 \\
 5 + \sqrt{49} = 12 \\
 5 + 7 = 12 \\
 12 = 12 \quad \checkmark
 \end{array}$$

**Example 2:** Solve for x:  $\sqrt{x+3}+8=6$ 

$$\sqrt{x+3} = -2$$

$$(\sqrt{x+3})^2 = (-2)^2$$

$$x+3 = 4$$

$$x = 1$$

but

$$\sqrt{1+3} + 8 = 6$$

$$2 + 8 = 6$$

$$10 \neq 6$$

No Answer**Example 3:** p. 297 - State restrictions, then solve for n:

$$n - \sqrt{5-n} = -7$$

$$-\sqrt{5-n} = -7 - n$$

$$(\sqrt{5-n})^2 = (7+n)^2$$

$$5-n \geq 0$$

$$-n \geq -5$$

$$n \leq 5$$

$$5-n = 49 + 14n + n^2$$

$$0 = 44 + 15n + n^2$$

$$n^2 + 15n + 44 = 0$$

$$(n+4)(n+11) = 0$$

$$n = -4 \text{ or } -11$$

$$n = -4$$

$$-4 - \sqrt{5 - (-4)} = -7$$

$$-4 - \sqrt{9} = -7$$

$$-4 - 3$$

✓

$$n = -11$$

$$-11 - \sqrt{5 - (-11)} = -7$$

$$-11 - \sqrt{16} = -7$$

$$-11 - 4 = -7$$

$$-15 = -7$$

No

$$\underline{n = -4 \text{ only}}$$

If an equation has two radicals, we may have to square twice in order to solve.

**Example 4:** State restrictions, and solve for x:

$$\sqrt{9x+1} - \sqrt{4x-2} = 3$$

$\sqrt{9x+1} \geq 0 \Rightarrow 9x+1 \geq 0 \Rightarrow x \geq -\frac{1}{9}$   
 $\sqrt{4x-2} \geq 0 \Rightarrow 4x-2 \geq 0 \Rightarrow x \geq \frac{1}{2}$

$(\sqrt{9x+1})^2 = (3 + \sqrt{4x-2})^2$

$$9x+1 = 9 + 6\sqrt{4x-2} + 4x-2$$

$$(5x-6)^2 = (6\sqrt{4x-2})^2$$

$$25x^2 - 60x + 36 = 36(4x-2)$$

$$25x^2 - 60x + 36 = 144x - 72$$

$$25x^2 - 204x + 108 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{7.6}{2} \text{ or } \frac{0.6}{2}$$

check on Calc.



check on Calc.



**Assignment:** p. 300 #3-7abc, 10ac, 12-15