

5.1 Working With Radicals

Radicals are roots, and are accepted as exact answers, instead of rounded decimals

e.g. $\sqrt{5}$, $\sqrt[3]{2}$, $\sqrt[3]{14x}$, $\sqrt[5]{16x^3y}$

A radical may have different parts:

$\nearrow$
root index

 $\sqrt[4]{50x}$

$\nwarrow$
root sign

 $\uparrow$
radicand

Because of even & odd numbers of negatives, we will have restrictions on the radicand, depending on the root index.

i.e. even roots cannot have a negative radicand

odd roots may have a neg. radicand, and will be negative.

Example 1: give any restrictions on the variable:

a) $\sqrt{2x+1}$

$$2x+1 \geq 0 \quad \rightarrow \quad x \geq -\frac{1}{2}$$

b) $\sqrt[3]{2x+1}$

No Restrictions

Just like fractions may be mixed, like $3\frac{3}{4}$ or Improper, like $\frac{15}{4}$,

radicals may also be mixed, like $2\sqrt{5}$ or Entire, like $\sqrt{20}$.

Mixed to Entire: Square 1st number, then multiply by the radicand.

Easy

$\rightarrow$ Cube if cube root

Example 2: express as entire radicals

a) $3\sqrt{6}$ $\sqrt{9}\sqrt{6}$
 $= \sqrt{54}$

b) $-7\sqrt{3}$ $-\sqrt{49}\sqrt{3}$
 $= -\sqrt{147}$

c) $2\sqrt[3]{5}$ $\sqrt[3]{8}\sqrt[3]{5}$
 $= \sqrt[3]{40}$

d) $5x\sqrt{6}$ $\sqrt{25x^2}\sqrt{6}$
 $= \sqrt{150x^2}$

e) $5\sqrt[3]{2}$ $\sqrt[3]{125}\sqrt[3]{2}$
 $= \sqrt[3]{250}$

f) $4\sqrt[4]{4}$ $\sqrt[4]{256}\sqrt[4]{4}$
 $= \sqrt[4]{1024}$

Example 3: Rearrange the radicals from smallest to largest:

$$\frac{1}{2}\sqrt{320}, 3\sqrt{10}, 5\sqrt{3}$$

$$\swarrow \quad \downarrow \quad \downarrow$$

$$\sqrt{80} \quad \sqrt{90} \quad \sqrt{75}$$

$$5\sqrt{3}, \frac{1}{2}\sqrt{320}, 3\sqrt{10}$$

Entire to Mixed:

Harder

Express as a product of perfect square and another radical, then simplify.

This means we need to recognize the perfect squares.

4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289,
324, 361, 400,

Example 4: Express as simplified mixed radicals

a) $\sqrt{20} \quad \sqrt{4} \sqrt{5}$

$$= 2\sqrt{5}$$

b) $\sqrt{72x^2} \quad \sqrt{36} \sqrt{2} \sqrt{x^2}$

$$= 6x\sqrt{2}$$

c) $\sqrt{320} \quad \sqrt{64} \sqrt{5}$

$$= 8\sqrt{5}$$

d) $\sqrt[3]{24}$

$$\sqrt[3]{8} \sqrt[3]{3}$$

$$= 2\sqrt[3]{3}$$

e) $\sqrt[3]{250}$

$$\sqrt[3]{125} \sqrt[3]{2}$$

$$= 5\sqrt[3]{2}$$

f) $\sqrt{216x^5}$

$$\sqrt{36} \sqrt{6} \sqrt{x^4} \sqrt{x}$$

$$= 6x^2\sqrt{6x}$$

Assignment: p. 279 # 1-6, 11, 13, 14