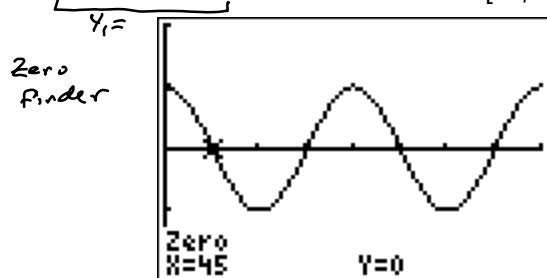


## Unit 5: Trigonometry & The Unit Circle

### 5.4 Equations & Graphs of Trigonometric Functions

Ex. Use your graphing calculator to determine the solutions for the trigonometric equation

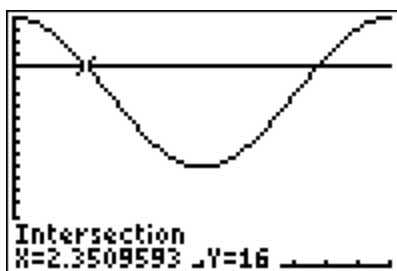
$2\cos^2 x - 1 = 0$  in the interval  $[0^\circ, 360^\circ]$ . Verify algebraically.



$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

$$\begin{aligned} 2\cos^2 x - 1 &= 0 \\ \cos^2 x &= \frac{1}{2} \\ \cos x &= \pm \frac{1}{\sqrt{2}} \\ \theta &= 45^\circ \\ 180 - 45^\circ &= 135^\circ \\ 180 + 45^\circ &= 225^\circ \\ 360 - 45^\circ &= 315^\circ \end{aligned}$$

Ex. Determine the general solutions for the trigonometric equation  $16 = 6\cos\frac{\pi}{6}x + 14$ . Express your answer to the nearest hundredth.



intersect  
finder

$$x = 2.35 + 12n \quad \text{and} \quad x = 9.65 + 12n \quad ; \quad n \in \mathbb{Z}$$

$n \in \mathbb{Z}$

Ex. The depth of water ( $d$  in meters) at dock by the Bay of Fundy at a certain time ( $t$  in hours after midnight) varies according to the function:

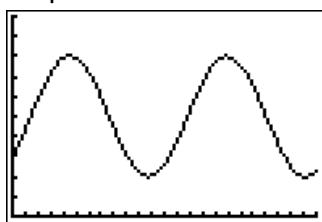
$$d(t) = 3\cos\frac{2\pi}{12.4}(t - 4.5) + 5$$

$$\begin{aligned} \text{amp} &= 3 \text{ m} & \text{change in depth} \\ &\quad \text{from the mean} \\ \text{vert. disp.} &= 5 \text{ m} & \text{the mean} \\ &\quad \left. \begin{array}{c} \text{low} \\ 2 \text{ m} \\ \text{high} \\ 8 \text{ m} \end{array} \right. \end{aligned}$$

At what time is the first low tide?

$$\text{high tide time} + \frac{1}{2} \text{ period} = 4.5 + 6.2 = 10.7 \text{ h} \quad 10:42 \text{ AM}$$

Graph in calculator for a 24 hour period.



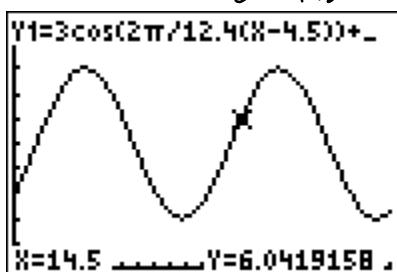
phase shift = 4.5 h *1st high tide (cosine)*  
at 4:30 AM

$$\text{period} = \frac{2\pi}{2\pi/12.4} = 2\pi \times \frac{12.4}{2\pi} = 12.4 \text{ h}$$

*time between high tides*

Find the depth at 2:30 PM to the nearest tenth.

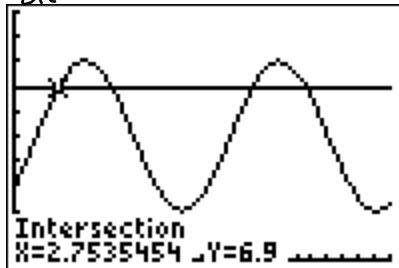
$$t = 14.5$$



$$6.0 \text{ m}$$

A ship can dock safely if the depth of water is at least 6.9 m. For how many hours in a 24 hour cycle is it safe to dock?

$$d(t) = 6.9$$



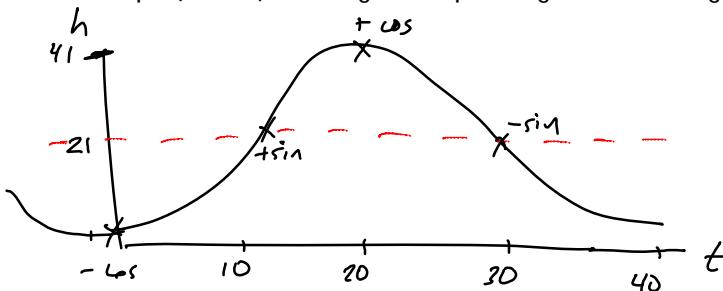
$$t = 2.75h$$

$$t = 6.25h$$

$$2(6.25 - 2.75) \\ 2(3.5) \\ = 7 \text{ hrs}$$

Ex. A Ferris wheel has a radius of 20 m. It rotates once every 40 seconds. Passengers get on at the lowest point 1 m above the ground. Determine a function that represents the height (h in meters) of a passenger at time (t in seconds) after it starts to rotate.

Graph (sketch) the height of a passenger above the ground for one rotation:



$$\text{amp} = 20$$

$$\text{period} = 40$$

$$\therefore b = \frac{2\pi}{40}$$

$$\text{vert. disp.} = 21$$

possible functions:

#1 + cosine  $h(t) = 20 \cos \frac{2\pi}{40}(t - 20) + 21$

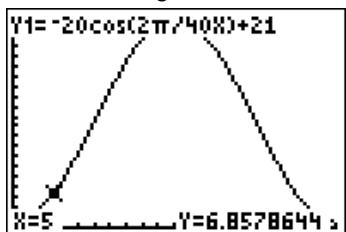
#2 - cosine  $h(t) = -20 \cos \frac{2\pi}{40}t + 21$

easiest to graph

#3 + sine  $h(t) = 20 \sin \frac{2\pi}{40}(t - 10) + 21$

#4 - sine  $h(t) = -20 \sin \frac{2\pi}{40}(t - 30) + 21$

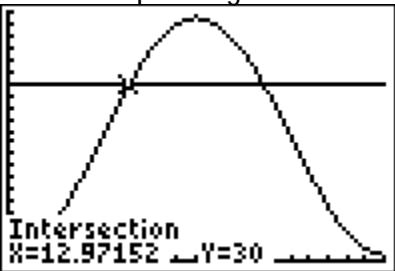
Find the height after 5 seconds? After 22 seconds? (nearest tenth)



$$h(t) = 6.9 \text{ m}$$

$$h(22) = 40.0 \text{ m}$$

When is a passenger at 30 m height during one rotation of the wheel?



$$t = 13.0 \text{ s}$$

$$\leftarrow 27.0 \text{ s}$$

Practice: pg. 275/# 1, 3 – 6, 8 – 11, 13, 15, 17 – 20