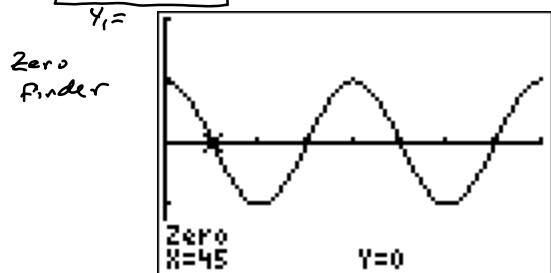


Unit 5: Trigonometry & The Unit Circle

5.4 Equations & Graphs of Trigonometric Functions

Ex. Use your graphing calculator to determine the solutions for the trigonometric equation

$2 \cos^2 x - 1 = 0$ in the interval $[0^\circ, 360^\circ]$. Verify algebraically.



$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

Algebraic solution:

$$2 \cos^2 x - 1 = 0$$

$$\cos^2 x = \frac{1}{2}$$

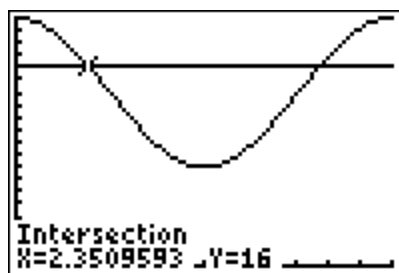
$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$\phi_k = 45^\circ$

$180 - 45 \quad 180 + 45 \quad 360 - 45$

$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

Ex. Determine the general solutions for the trigonometric equation $16 = 6 \cos \frac{\pi}{6} x + 14$. Express your answer to the nearest hundredth.



intersect
finder

$$x = 2.35 + 12n$$

$$x = 9.65 + 12n \quad ; n \in \mathbb{I}$$

$n \in \mathbb{Z}$

period = $\frac{2\pi}{\pi/6}$

$$= 2\pi \times \frac{6}{\pi}$$

$$= 12$$

Ex. The depth of water (d in meters) at dock by the Bay of Fundy at a certain time (t in hours after midnight) varies according to the function:

$$d(t) = 3 \cos \frac{2\pi}{12.4} (t - 4.5) + 5$$

amp = 3 m change in depth from the mean

vert. disp. = 5 m the mean

low 2m
high 8m

phase shift = 4.5 h 1st high tide (cosine) at 4:30 AM

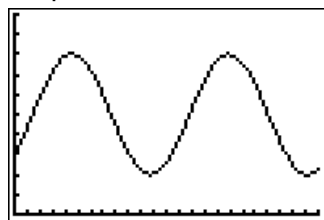
period = $\frac{2\pi}{2\pi/12.4} = 2\pi \times \frac{12.4}{2\pi} = 12.4$ h

time between high tides

At what time is the first low tide?

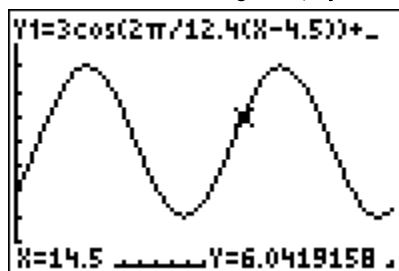
high tide time + $\frac{1}{2}$ period = $4.5 + 6.2 = 10.7$ h 10:42 AM

Graph in calculator for a 24 hour period.



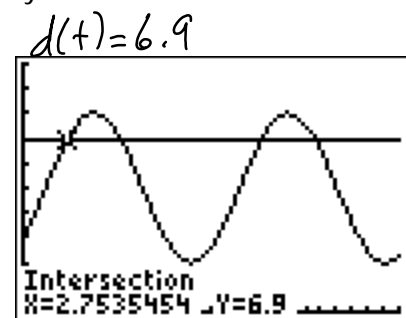
Find the depth at 2:30 PM to the nearest tenth.

$t = 14.5$



$$6.0 \text{ m}$$

A ship can dock safely if the depth of water is at least 6.9 m. For how many hours in a 24 hour cycle is it safe to dock?



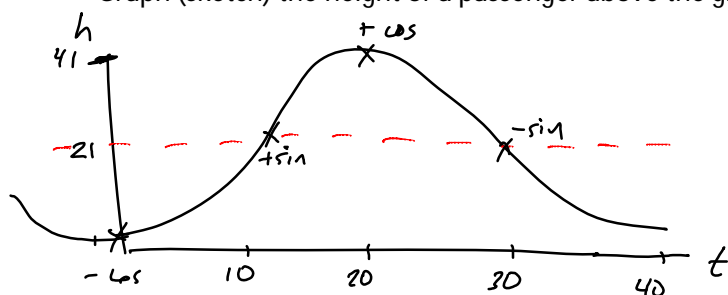
$$t = 2.75h$$

$$t = 6.25h$$

$$\begin{aligned} & 2(6.25 - 2.75) \\ & 2(3.5) \\ & = \boxed{7 \text{ hrs}} \end{aligned}$$

Ex. A Ferris wheel has a radius of 20 m. It rotates once every 40 seconds. Passengers get on at the lowest point 1 m above the ground. Determine a function that represents the height (h in meters) of a passenger at time (t in seconds) after it starts to rotate.

Graph (sketch) the height of a passenger above the ground for one rotation:



$$\text{amp} = \underline{20}$$

$$\text{period} = \underline{40}$$

$$\therefore b = \underline{\frac{2\pi}{40}}$$

$$\text{vert. disp.} = \underline{21}$$

possible functions:

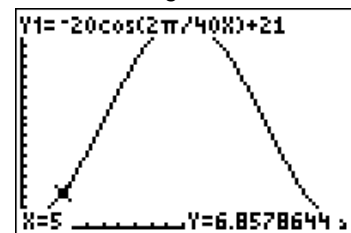
#1 + cosine $h(t) = 20 \cos \frac{2\pi}{40}(t - 20) + 21$

#2 - cosine $h(t) = -20 \cos \frac{2\pi}{40}t + 21$ easiest to graph

#3 + sine $h(t) = 20 \sin \frac{2\pi}{40}(t - 10) + 21$

#4 - sine $h(t) = -20 \sin \frac{2\pi}{40}(\underbrace{t - 30}_{t + 10}) + 21$

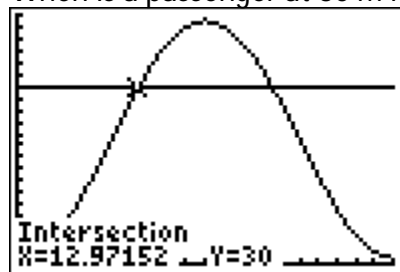
Find the height after 5 seconds? After 22 seconds? (nearest tenth)



$$h(5) = 6.9m$$

$$h(22) = 40.0m$$

When is a passenger at 30 m height during one rotation of the wheel?



$$t = 13.0s$$

$$t = 27.0s$$