PreCalculus 11

Name______________
Date______________

4.4 The Quadratic Formula

Some quadratic equations can be solved by factoring, e.g. x^2+5x+6, but some cannot - $x^2+90x+17$.

We can however, complete the square, then use Algebra to solve!.

Example 1: give the exact answers to: $3x^2+6x-8=0$

1. Move 8 over to right side $3x^2+6x=+8$
2. Factor out the 3 $3(x^2+2x+\_\_)=+8$
3. Add ± 1 to complete the square $3(x^2+2x+1-1)=+8$
4. Write perfect square $(x+1)^2$ $3(x+1)^2-3=+8$ $\quad +3 \quad +3$
5. Move -3 to right side $3(x+1)^2=11$
6. Divide both sides by 3 $(x+1)^2=\frac{11}{3}$
7. Take square root of both sides $x+1=\pm\sqrt{\frac{11}{3}}$
8. Move 1 to right side $\boxed{x=-1\pm\sqrt{\frac{11}{3}}}$

We can go through these steps for any equation in standard form (________________________),
so what if we performed all these operations without numerical values?

Example 2: give the exact answers to: $ax^2 + bx + c = 0$

1. Move c over to right side $\quad ax^2 + bx = -c$
2. Factor out the a $\quad x^2 + \frac{b}{a}x = -\frac{c}{a}$ $\quad -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} = \frac{-4ac + b^2}{4a^2}$
3. Add $\left(\frac{b}{2a}\right)^2 \rightarrow \frac{b^2}{4a^2}$ to complete the square $\quad x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \underbrace{-\frac{c}{a} + \frac{b^2}{4a^2}}$
4. Write perfect square $\left(x + \frac{b}{2a}\right)^2$ $\quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$
5. Move ________ to right side $\quad x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$
6. Divide both sides by ________ $\quad x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
7. Take square root of both sides
8. Move ________ to right side $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The Quadratic Formula: for any equation $ax^2 + bx + c = 0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example 3: Solve for x (to the nearest hundredth)

a) $x^2 + 2x = 15 \rightarrow x^2+2x-15=0$

$a=1$

$b=2$

$c=-15$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(-15)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4+60}}{2}$$

$$x=\frac{-2\pm\sqrt{64}}{2}$$

$$x=\frac{-2\pm 8}{2}$$

$$\rightarrow \frac{-2+8}{2}=\frac{6}{2}=\boxed{3}$$

$$\rightarrow \frac{-2-8}{2}=\frac{-10}{2}=\boxed{-5}$$

b) $4x^2 - 7x - 1 = 0$

$a=4$

$b=-7$

$c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(4)(-1)}}{2(4)}$$

$$x=\frac{7\pm\sqrt{49+16}}{8}$$

$$\boxed{x=\frac{7\pm\sqrt{65}}{8}}$$

c) $\left(2x^2 + \frac{x}{3} = \frac{5}{2}\right) \times 6$

$12x^2 + 2x = 15$

$12x^2 + 2x - 15 = 0$

$a = 12$

$b = 2$

$c = -15$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(12)(-15)}}{2(12)}$$

$$x = \frac{-2 \pm \sqrt{4 + 720}}{24}$$

$\Rightarrow$ simplify!

$$x = \frac{-2 \pm \sqrt{724}}{24}$$

$$x = \frac{-2 \pm \sqrt{4 \cdot 181}}{24} = \frac{-2 \pm 2\sqrt{181}}{24}$$

$$\boxed{x = \frac{-1 \pm \sqrt{181}}{12}}$$

d) $3x(2x + 1) + 2 = -2x$

$6x^2 + 3x + 2 = -2x$

$+2x \qquad +2x$

$6x^2 + 5x + 2 = 0$

$a = 6$

$b = 5$

$c = 2$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(6)(2)}}{2(6)}$$

$$x = \frac{-5 \pm \sqrt{25 - 48}}{12}$$

$$\boxed{x = \frac{-5 \pm \sqrt{-23}}{12}}$$

Not possible b/c $\sqrt{-23}$ does not exist!

Assignment: Day 1 – p. 254# 3-5(abcd), 8