PreCalculus 11

Name____________
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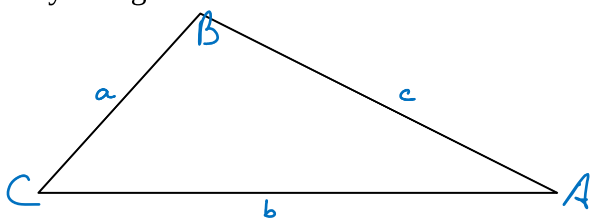
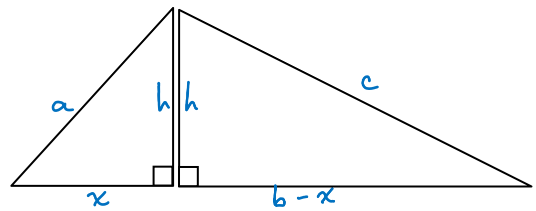
2.4 The Cosine Law

Once again, start with any triangle ΔABC:



If we drop a perpendicular and create two separate triangles:



Because these are right triangles, we can use the Pythagorean Theorem:

$$x^2 + h^2 = a^2 \qquad \& \qquad (b-x)^2 + h^2 = c^2$$

$$h^2 = a^2 - x^2 \qquad\qquad h^2 = c^2 - (b-x)^2$$

$$\therefore \quad a^2 - x^2 = c^2 - (b-x)^2 \qquad \text{as both} = h^2 \quad ☺$$

Expanding out the binomial term,

$$a^2 - x^2 = c^2 - (b^2 - 2bx + x^2)$$

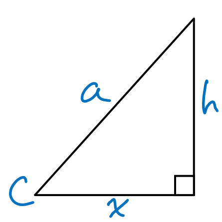
$$a^2 - x^2 = c^2 - b^2 + 2bx - x^2$$

$$a^2 = c^2 - b^2 + 2bx$$

Isolating the c^2 term,

$$a^2 + b^2 - 2bx = c^2$$

We still need to replace the x term. If we consider the left side right triangle,



$$\cos C = \frac{x}{a}$$

$$\therefore \quad x = a \cos C$$

and the formula becomes:

$$c^2 = a^2 + b^2 - 2ba \cos C$$

The Cosine Law (for any ΔABC)

Pythagoras for any △

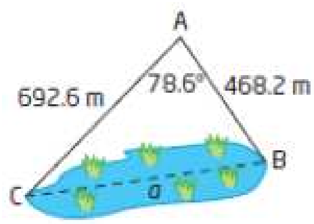
$c^2 = a^2 + b^2 - 2ab\cos C$

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

Example 1: p.116 - Determine a Distance

A surveyor needs to find the length of a swampy area near Fishing Lake, Manitoba. The surveyor sets up her transit at a point A. She measures the distance to one end of the swamp as 468.2 m, the distance to the opposite end of the swamp as 692.6 m, and the angle of sight between the two as 78.6°. Determine the length of the swampy area, to the nearest tenth of a metre.



$a^2 = b^2 + c^2 - 2bc\cos A$

$= (692.6)^2 + (468.2)^2 - 2(692.6)(468.2)\cos(78.6°)$

$= 570\,715.2$

$\therefore\ a = \sqrt{570\,715.2}$

$= 755.5\text{ m}$

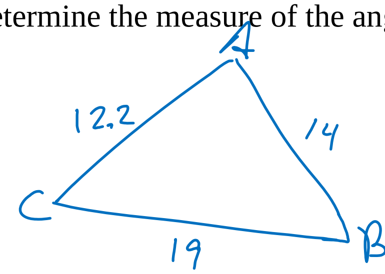
Remember the different classifications of triangles? (ASA, SSA, . . .)

Example 1 was an SAS triangle. We couldn't use the Sine Law as we didn't have an opposite side and angle.

Another kind of triangle we can't use the Sine Law with is an SSS - all 3 sides but no angles. (don't know opp side & angle)

Example 2: p.117 Determine an Angle:

The Lions' Gate Bridge has been a Vancouver landmark since it opened in 1938. It is the longest suspension bridge in Western Canada. The bridge is strengthened by triangular braces. Suppose one brace has side lengths 14 m, 19 m, and 12.2 m. Determine the measure of the angle opposite the 14-m side, to the nearest degree.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$14^2 = 19^2 + 12.2^2 - 2(19)(12.2)\cos C$$

$$196 = 509.84 - 463.6\cos C$$

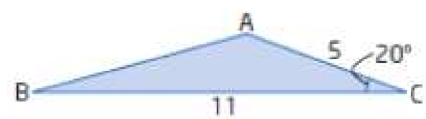
don't try to combine these two! ever!

$$-313.84 = -463.6\cos C$$

$$0.67696 = \cos C$$

$$\angle C = 47^\circ$$

Example 3: p. 118 Solve a Triangle



must start with Cos Law:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$= 11^2 + 5^2 - 2(11)(5)\cos 20^\circ$$

$$= 42.63$$

$$\therefore\ c = 6.5$$

$\angle A = 144^\circ$ $a = 11$

$\angle B = 16^\circ$ $b = 5$

$\angle C = 20^\circ$ $c = 6.3$

Now, the Sine Law

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin B}{5} = \frac{\sin 20^\circ}{6.3}$$

$$\angle B = 16^\circ \rightarrow \therefore \angle A = 144^\circ$$

Assignment: p. 119 # 1-3, 7, 10