

8.2 Systems of Equations – Substitution

Remember from last year, we can solve systems of equations graphically, or algebraically by using Substitution or elimination.

To solve by *substitution*

- isolate x or y in one of the equations
- substitute into the other equation
- solve for x or y
- solve for y or x \rightarrow sub into the easier eq \simeq
- check answers in both equations

Example 1: p. 441 - Solve the **Linear-Quadratic** system:

$$a) \quad 5x - y = 10 \quad \text{and} \quad x^2 + x - 2y = 0$$

$$\begin{aligned}
 & \downarrow \\
 y = 5x - 10 & \rightarrow x^2 + x - 2(5x - 10) = 0 \\
 & x^2 + x - 10x + 20 = 0 \\
 & x^2 - 9x + 20 = 0 \\
 & (x-5)(x-4) = 0 \\
 \therefore y = 5(5) - 10 & \quad x = 5 \text{ or } 4 \\
 \text{or} \\
 y = 5(4) - 10 & \quad \swarrow \quad \overline{x=5} \text{ or } \overline{x=4} \\
 & (5, 15) \text{ & } (4, 10)
 \end{aligned}$$

b) verify your solution

$$\begin{aligned} 5(s) - 15 &= 10 \\ 25 - 15 &= 10 \\ 10 &= 10 \end{aligned}$$

$$\begin{aligned}
 5^2 + 5 - 2(15) &= 0 \\
 25 + 5 - 30 &= 0 \\
 30 - 30 &= 0 \\
 0 &= 0
 \end{aligned}$$

$$5(4) - 10 = 10$$
$$20 - 10 = 10$$
$$10 = 10$$

$$\begin{array}{rcl}
 4^2 + 4 - 2(10) & = 0 \\
 16 + 4 - 20 & = 0 \\
 20 - 20 & = 0 \\
 1 & 0 & = 0 \quad \checkmark
 \end{array}$$

$$\underline{(4, 10)}$$

Notice that we could have solved for $x = \frac{10+y}{5}$, but this would have resulted in the equation $\underline{(2+\frac{y}{5})^2 + (2+\frac{y}{5}) - 2y = 0}$ *Too ugly!*

Example 2: p. 443 - Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased by three times the larger, the result is 93.

a) Write a system of equations that relates to the problem.

$$\begin{aligned} x &= \text{smaller integer} \\ y &= \text{larger integer} \end{aligned}$$

$$\begin{aligned} x+2y &= 46 \\ &\hline & x^2 - 3y &= 93 \\ &\hline \end{aligned}$$

b) Solve the system algebraically:

$$\begin{aligned} \text{isolate } x &= 46-2y \\ \xrightarrow{\quad} & (46-2y)^2 - 3y = 93 \end{aligned}$$

$$2116 - 184y + 4y^2 - 3y = 93$$

$$4y^2 - 187y + 2023 = 0$$

\downarrow Quadratic Formula

$$y = 29.75 \text{ or } y = 17$$

Not an integer!

$$\begin{aligned} \therefore y &= 17 & \& x+2(17) &= 46 \\ &\hline & x+34 &= 46 \\ &\hline & x &= 12 \end{aligned}$$

The two integers are 12 & 17

Assignment: p. 451 # 2, 3, 8, 9