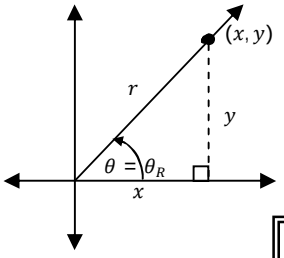
PreCalculus 12 Date _____

Unit 4: Trigonometry & The Unit Circle

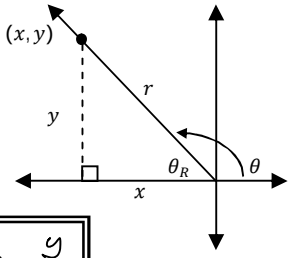
4.3 Trigonometric Ratios

To determine **primary trigonometric ratios** (sine, cosine, tangent) for standard position angle θ we use a point (x, y) on the terminal arm and the right triangle containing the reference angle (reference triangle).

Can be done in any quadrant – use the reference Δ to state the ratios



$r = radius$
$r = \sqrt{x^2 + y^2}$



$\therefore$ $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

If $P(\theta) = (x, y)$ is the point on the terminal arm of angle θ that intersects the unit circle then:

$r =$ 1 $\sin\theta = \frac{y}{1} = y$ $\cos\theta = \frac{x}{1} = x$

Thus, $P(\theta) =$ $(\cos\theta, \sin\theta)$ on the unit circle.

Reciprocal Trig Ratios:

Cosecant $\csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp} = \frac{r}{y}$

Secant $\sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj} = \frac{r}{x}$

Cotangent $\cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp} = \frac{x}{y}$

Ex. The point A $\left(-\frac{3}{5}, -\frac{4}{5}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle θ in standard position. Determine the values of the six trigonometric ratios for θ in lowest terms.

because θ is in the unit circle, $P(\theta) = (\cos\theta, \sin\theta) = (-3/5, -4/5)$

$\sin\theta = -4/5$ $\csc\theta = -5/4$

$\cos\theta = -3/5$ $\sec\theta = -5/3$

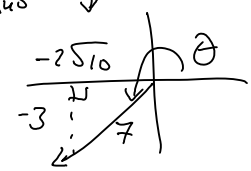
$\tan\theta = y/x = \frac{-4/5}{-3/5} = -4/5 \times -5/3 = 4/3$ $\cot\theta = 3/4$

Ex. If $\csc\theta = -\frac{7}{3}$ for angle θ in standard position in quadrant III, determine the exact values of the 5 other trigonometric ratios.

$-\sqrt{7^2 - (-3)^2}$

$-\sqrt{40}$

$-2\sqrt{10}$



$\csc\theta = -7/3 \longrightarrow \sin\theta = -3/7$

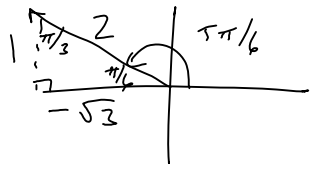
$\cos\theta = -2\sqrt{10}/7$ $\sec\theta = -7/2\sqrt{10}$

$\tan\theta = 3/2\sqrt{10}$ $\cot\theta = 2\sqrt{10}/3$

Exact Values of Trigonometric Ratios

Ex. Determine the exact value for each. Draw diagrams to illustrate your answers.

a) $\cos\frac{5\pi}{6}$



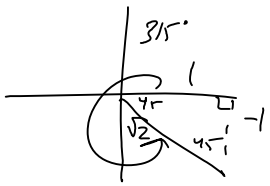
QII

$\theta_R = \pi - 5\pi/6$

$= \pi/6$

$\cos 5\pi/6 = \boxed{-\sqrt{3}/2}$

c) $\sec 315°$



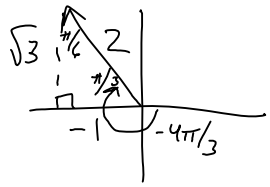
QIV

$\theta_R = 360 - 315°$

$= 45°$

$\sec 315° = \boxed{\sqrt{2}}$

b) $\sin\left(-\frac{4\pi}{3}\right)$



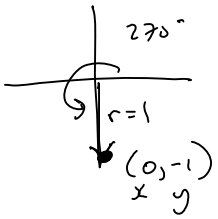
QII

$\theta_R = 4\pi/3 - \pi$

$= \pi/3$

$\sin\left(-4\pi/3\right) = \boxed{\sqrt{3}/2}$

d) $\cot 270°$



$\cot\theta = \frac{x}{y}$

$\cot 270° = \frac{0}{-1}$

$= \boxed{0}$

Approximate Values of Trigonometric Ratios

Ex. Determine the approximate value for each trig ratio, to four decimal places.

a) $\cot\frac{7\pi}{5}$

$\cot\theta = \frac{1}{\tan\theta}$

$= \frac{1}{\tan 7\pi/5}$

$= \boxed{0.3249}$

b) $\sec 4.2$

$= \frac{1}{\cos 4.2}$

$= \boxed{-2.0397}$

c) $2\csc(-70°)$

* MODE!

$= 2 \times \frac{1}{\sin(-70°)}$

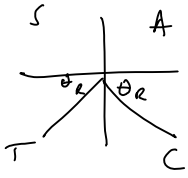
$= \frac{2}{\sin(-70°)}$

$= \boxed{-2.1284}$

Finding Angles Given Their Trigonometric Ratios

Ex. Determine the measures of all angles that satisfy the following. Use diagrams in your explanation. Give exact answers where possible, round to the nearest tenth where necessary.

a) $\sin\theta = -0.366$, $0 \le \theta < 360°$



QIII + QIV

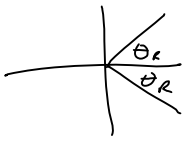
$\theta_R = \sin^{-1}(0.366)$

$= 21.5°$

$180 + 21.5$ $\quad$ $360 - 21.5$

$\theta = \boxed{201.5° \text{ \& } 338.5°}$

b) $\cos\theta = 0.879$, $0° \le \theta < 2\pi$ ← RADS!



QI + QIV

$\theta_R = \cos^{-1}(0.879)$

$= 0.5$

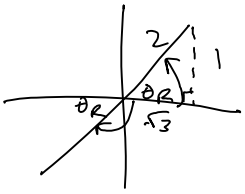
$2\pi - 0.5$

$\theta = \boxed{0.5,\ 5.8}$

c) $\cot\theta = \frac{\sqrt{3}}{1}$, $-180° \le \theta < 180°$

QI + QIII

$\tan\theta = \frac{1}{\sqrt{3}}$



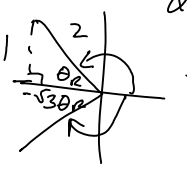
$\therefore \theta_R = 30°$

$\boxed{30°,\ -150°}$

d) $\sec\theta = -\frac{2}{\sqrt{3}}$, $-2\pi \le \theta < 2\pi$

QII + QIII

$\cos\theta = -\frac{\sqrt{3}}{2}$



$\theta_R = \pi/6$

$\pi - \pi/6$ $\quad$ $\pi + \pi/6$

$\boxed{5\pi/6 \quad 7\pi/6 \quad -5\pi/6 \quad -7\pi/6}$

Practice: pg. 201/# 1 – 6, 8 – 14, 19, C2, C4