PreCalculus 11

Name Key
Date

4.3 Solving by Completing the Square

Sometimes factoring a quadratic equation is not the most efficient way to solve. Consider an equation containing a difference of squares:

$x^2 - 25 = 0$ by factoring:

$(x+5)(x-5) = 0$

$x = 5$ or -5

$x^2 - 25 = 0$ by square root:

$x^2 = 25$

$x = \pm\sqrt{25} = \pm 5$

If we remember to use ~~exam~~ Algebra, we can solve by equation properties and by using a square root.

Example 1: Solve for *x*:

a) $\frac{x^2}{3} - 1 = 11$

$\frac{x^2}{3} = 12$

$x^2 = 36$ $\quad x = \pm\sqrt{36}$ $\quad \pm 6$

b) $(x-3)^2 = 25$

$x - 3 = \pm 25$

$+3 \quad +3$

$x = 3 \pm 25$

$x = 28$ or -22

What if the equation doesn't have a perfect square? We may need to Complete the Square before introducing a Square root to solve.

Remember, to find the value to add & subtract, $\left(\frac{b}{2}\right)^2$.

Example 2: Solve for *x*:

a) $x^2 - 10x = -16$

$(x^2 - 10x + 25 - 25) + 16 = 0$

$(x^2 - 10x + 25) - 9 = 0$

$(x-5)^2 - 9 = 0$

$(x-5)^2 = 9 \qquad x - 5 = \pm 3$

$x = 5 \pm 3$

$x = 8$

$x = 2$

b) $5x^2 + 20x + 7 = 0$ to 2 decimal places, then also as exact values

$5(x^2+4x+\_\_) + 7 = 0$

$5(x^2+4x+4-4) + 7 = 0$

$5(x^2+4x+4) - 13 = 0$

$5(x+4)^2 = 13$

$(x+4)^2 = \frac{13}{5}$

$x+4 = \pm\sqrt{\frac{13}{5}}$

$x = -4 \pm \sqrt{\frac{13}{5}}$

$x = -4 + \sqrt{\frac{13}{5}} = -2.39$

$x = -4 - \sqrt{\frac{13}{5}} = -5.61$

Why not simply factor the last equation? not easy/not possible to factor.

We may need to simplify the radicals with exact answers:

$\sqrt{60} \rightarrow$ ~~$\sqrt{10}\sqrt{6}$~~

$\sqrt{4 \cdot 15} = \sqrt{4}\sqrt{15} = 2\sqrt{15}$

4, 9, 16

Example 3: Solve and leave in simplest radical form:

a) $x^2 = 20x - 4$

$x^2 - 20x + 4 = 0$

$(x^2 - 20x + \_\_ - \_\_) + 4 = 0$

$x^2 - 20x + 100 - 100 + 4 = 0$

$(x-10)^2 - 96 = 0$

$x - 10 = \pm\sqrt{96}$

$x = 10 \pm \sqrt{96}$

$x = 10 \pm \sqrt{16 \cdot 6}$

$x = 10 \pm 4\sqrt{6}$

b) $3x^2 - 6x = -2$

$3(x^2 - 2x + 1 - 1) = -2$

$3(x^2 - 2x + 1) - 3 = -2$

$3(x-1)^2 = 1$

$(x-1)^2 = \frac{1}{3}$

$(x-1) = \pm\sqrt{\frac{1}{3}}$

$x = 1 \pm \sqrt{\frac{1}{3}} = 1 \pm \frac{\sqrt{1}}{\sqrt{3}} = \boxed{1 \pm \frac{1}{\sqrt{3}}}$

Example 4: Solve to 2 decimal places:

$x = 1 + \frac{1}{\sqrt{3}} = 1.58$

$x = 1 - \frac{1}{\sqrt{3}} = 0.42$

Assignment: p. 240 # 1, 3-7(abcd), 10, 11