

Unit 1: Function Transformations

1.3 Combining Transformations

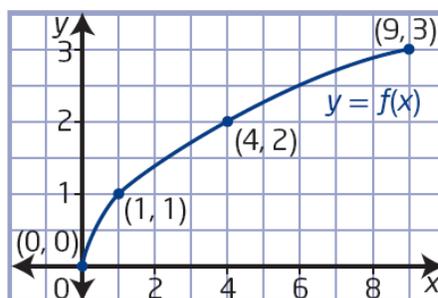
$$y - k = af(b(x - h)) \text{ OR } y = af(b(x - h)) + k$$

When applying multiple transformations the shape/orientation of the graph will be changed before its position.

- ie. } BEDMAS!
- stretches & reflections – multiplication
 - before
 - translations – addition/subtraction

* also very important to factor the coefficient of x inside the function

Ex. Given the graph of $f(x)$ shown describe the transformations that will be applied for each stated transformed function. Give the mapping notation for each and sketch the transformed graph.



a) $y = 3f(2x)$

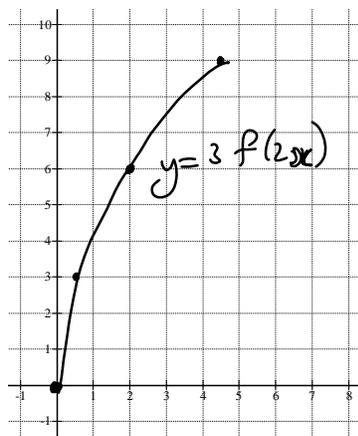
vertical expansion
by factor of 3

horizontal compression
by factor of $\frac{1}{2}$

$(x, y) \rightarrow (\frac{1}{2}x, 3y)$

| x | y |
|-----|-----|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |

| $\frac{1}{2}x$ | $3y$ |
|----------------|------|
| 0 | 0 |
| $\frac{1}{2}$ | 3 |
| 2 | 6 |
| $\frac{9}{2}$ | 9 |



b) $y = -f(3x + 6) + 2$

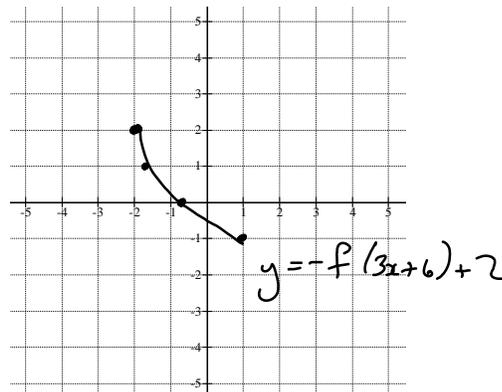
$= -f(3(x+2)) + 2$

- reflection in x -axis / vertical reflection
- horizontal compression by a factor of $\frac{1}{3}$
- translated horiz. 2 units left
- translated vert. 2 units up

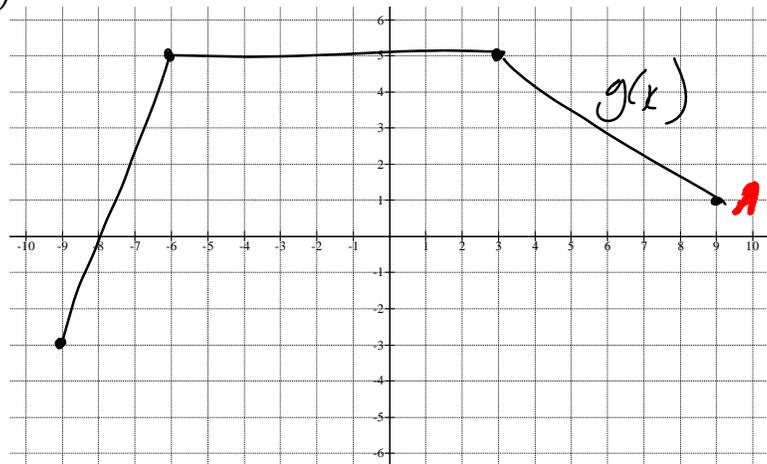
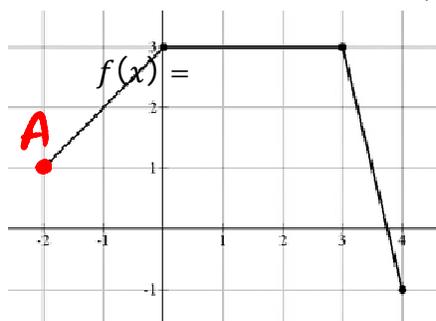
$(x, y) \rightarrow (\frac{1}{3}x - 2, -y + 2)$

| x | y |
|-----|-----|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |

| $\frac{1}{3}x - 2$ | $-y + 2$ |
|--------------------|----------|
| -2 | 2 |
| $-\frac{5}{3}$ | 1 |
| $-\frac{2}{3}$ | 0 |
| 1 | -1 |



Ex. Given the graph of $f(x)$ shown describe the transformations that will be applied, state the mapping notation and sketch $g(x) = 2f\left(1 - \frac{1}{3}x\right) - 1 = 2f\left(-\frac{1}{3}(x-3)\right) - 1$



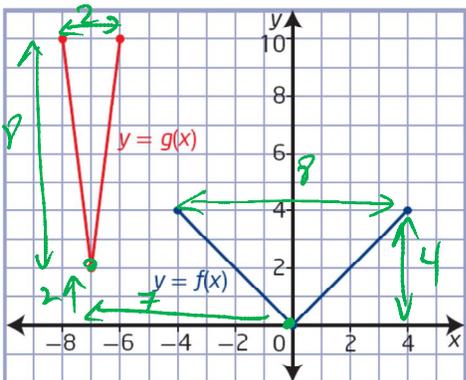
- Vertical expansion by a factor of 2
- reflection in the y-axis
- horizontal expansion by a factor of 3
- translated horiz. 3 units right
- translated vert. 1 unit down

$(x, y) \rightarrow (-3x + 3, 2y - 1)$
 $f(x) \quad 2y$
 $\begin{array}{c|c} -2 & 1 \\ 0 & 3 \\ 3 & 3 \\ 4 & -1 \end{array} \quad \begin{array}{c|c} -3x+3 & 2y-1 \\ 9 & 1 \\ 3 & 5 \\ -6 & 5 \\ -9 & -3 \end{array}$

Ex. The function $f(x) = x^2$ is expanded vertically by a factor of 5, reflected in the x-axis, compressed horizontally by a factor of $\frac{1}{2}$, translated 6 units right and 3 units down. Write the equation of the new function $g(x)$. $(x, y) \rightarrow (\frac{1}{2}x + 6, -y - 3)$

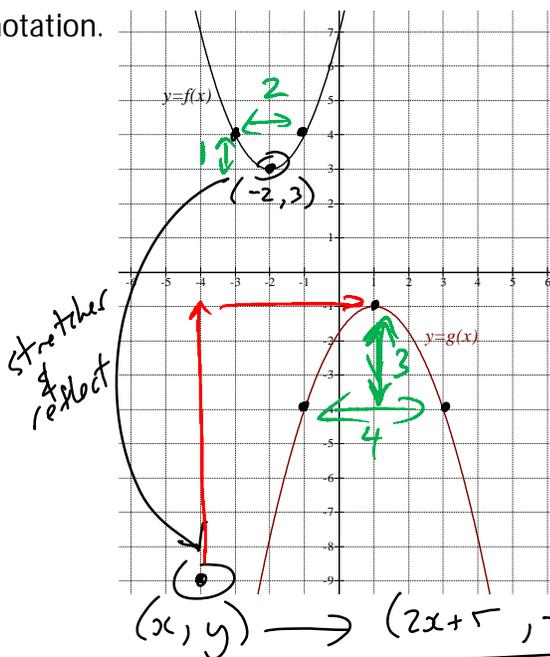
$$g(x) = -5f(2(x-6)) - 3 \quad \text{OR} \quad g(x) = -5(2(x-6))^2 - 3$$

Ex. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in function notation.



$$(x, y) \rightarrow (\frac{1}{2}x - 7, 2y + 2)$$

$$g(x) = 2f(4(x+7)) + 2$$



$$(x, y) \rightarrow (2x + 5, -3y + 8)$$

$$g(x) = -3f(\frac{1}{2}(x-5)) + 8$$