

1.3 Limits and Algebra

Name \_\_\_\_\_

Date \_\_\_\_\_

Remember, when evaluating limits, first plug in the number :

- if answer is number, this is the limit!
- if answer is  $\frac{0}{0}$ , this is an Indeterminate Form

To find an Indeterminate Form, we need to cancel out the 0 in denominator which may involve factoring (like yesterday), combining fractions to eliminate denominators, or combining radicals (Rationalize the denom or numerator)

**Example 1:** find  $\lim_{x \rightarrow -5} \left( \frac{2x^2 + 11x + 5}{x^2 - 25} \right)$   $\frac{50 + -55 + 5}{25 - 25} = \frac{0}{0} !$

$$= \lim_{x \rightarrow -5} \left( \frac{(2x+1)(x+5)}{(x-5)(x+5)} \right)$$

$$= \lim_{x \rightarrow -5} \left( \frac{2x+1}{x-5} \right) = \frac{-9}{-10} = \underline{\underline{\frac{9}{10}}}$$

**Example 2:** find  $\lim_{x \rightarrow 0} \left( \frac{\frac{1}{x+6} - \frac{1}{6}}{x} \right)$   $= \frac{\frac{1}{6} - \frac{1}{6}}{0} = \frac{0}{0}$

$$= \lim_{x \rightarrow 0} \left( \frac{\frac{1}{x+6} \left( \frac{6}{6} \right) - \frac{1}{6} \left( \frac{x+6}{x+6} \right)}{x} \right)$$

$$= \lim_{x \rightarrow 0} \left( \frac{\frac{6}{6x+36} - \frac{x+6}{6x+36}}{x} \right)$$

$$= \lim_{x \rightarrow 0} \left( \frac{-x}{6x+36} \right) \left( \frac{1}{x} \right) = \lim_{x \rightarrow 0} \frac{-1}{6x+36} = \underline{\underline{\frac{-1}{36}}}$$

**Example 3:** find  $\lim_{x \rightarrow 0} \frac{\sqrt{x+6} - \sqrt{6}}{x}$        $\frac{\sqrt{6} - \sqrt{6}}{0} = \frac{0}{0}$

Rationalize the numerator ?

$$= \lim_{x \rightarrow 0} \frac{\sqrt{x+6} - \sqrt{6}}{x} \left( \frac{\sqrt{x+6} + \sqrt{6}}{\sqrt{x+6} + \sqrt{6}} \right)$$

$$= \lim_{x \rightarrow 0} \left( \frac{x+6 - 6}{x(\sqrt{x+6} + \sqrt{6})} \right)$$

$$= \lim_{x \rightarrow 0} \left( \frac{x}{x(\sqrt{x+6} + \sqrt{6})} \right)$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+6} + \sqrt{6}}$$

$$= \frac{1}{\sqrt{6} + \sqrt{6}} \rightarrow \frac{1}{2\sqrt{6}} \quad \text{or} \quad \frac{\sqrt{2}}{12}$$

same decimal, so does it really matter if we rationalize the denominator ?