

8.3 Systems of Equations – Elimination

Remember from last year, systems can be solved algebraically by adding or subtracting multiples of the equations in order to eliminate one of the variables.

e.g. $2x + 5y + 19 = 0$ and $4x + 11y - 12 = 0$

$\xrightarrow{\times 2}$

$\underline{4x + 10y + 38 = 0}$

$1y - 50 = 0$

$y = 50$

Why not simply solve by substitution?

both equations would result in ugly fractions

Once we have eliminated one of the variables, we can solve for the other and then sub back in to find the other variable. The most important factor here is that one of the variables must be completely eliminated!

Example 1: p. 442 - Solve the system

$$\begin{array}{rcl} 5x - y = 10 & \text{and} & x^2 + x - 2y = 0 \\ \swarrow \text{ } x-2 & & + \quad -10x + 2y = -20 \\ & & \hline & & x^2 - 9x = -20 \end{array}$$

$$x^2 - 9x + 20 = 0$$

$$(x-4)(x-5) = 0$$

$$x = \underline{4} \text{ or } \underline{5}$$

$$\begin{aligned} 5x - y &= 10 \\ 20 - y &= 10 \\ -y &= -10 \\ y &= 10 \end{aligned}$$

$(4, 10)$

$$\begin{aligned} 5x - y &= 10 \\ 25 - y &= 10 \\ -y &= -15 \\ y &= 15 \end{aligned}$$

$(5, 15)$

Check

$$\begin{aligned} 5^2 + 5 - 2(15) &= 0 \\ 25 + 5 - 30 &= 0 \\ 30 - 30 &= 0 \end{aligned}$$



check

$$\begin{aligned} 4^2 + 4 - 2(10) &= 0 \\ 16 + 4 - 20 &= 0 \\ 20 - 20 &= 0 \end{aligned}$$

Why were we forced to eliminate the y variable?

did not have x^2 in both eqns

Example 2: p. 447 $3x^2 - x - y - 2 = 0$ and $6x^2 + 4x - y = 4$

$$\rightarrow \underline{3x^2 - x - y = 2}$$

$$3x^2 + 5x = 2$$

$$3x^2 + 5x - 2 = 0$$

$$3x^2 + 6x - 1x - 2 = 0$$

$$3x(x+2) - 1(x+2) = 0$$

$$(x+2)(3x-1) = 0$$

$$x = \underline{-2} \text{ or } \underline{\frac{1}{3}}$$

$$3(-2)^2 - (-2) - y - 2 = 0$$

$$12 + 2 - y - 2 = 0$$

$$12 - y = 0$$

$$y = 12$$

$$\underline{(-2, 12)}$$

Check $6(-2)^2 + 4(-2) - (-12) = 4$

$$24 - 8 - 12 = 4$$

$$24 - 20 = 4$$

$$4 = 4 \quad \checkmark$$

$$3\left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) - y - 2 = 0$$

$$3\left(\frac{1}{9}\right) - \frac{1}{3} - y = 2$$

$$\frac{1}{3} - \frac{1}{3} - y = 2$$

$$-y = 2$$

$$\underline{\underline{\left(\frac{1}{3}, -2\right)}}$$

Check $6\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right) - (-2) = 4$

$$\frac{6}{9} + \frac{4}{3} + 2 = 4$$

$$\frac{2}{3} + \frac{4}{3} + 2 = 4$$

$$2 + 2 = 4 \quad \checkmark$$

Key Ideas – p. 451 To solve a system using elimination:

- rearrange the equations so that like terms line up
- multiply one or both equations by a constant to equate one of the variables
- add or subtract to eliminate a variable and solve for the remaining one
- substitute back in to determine the other variable
- Verify your answers by substituting into original eq's

Assignment: p. 452 # 4, 10, 11, 21