Name______________
Date_______________

1.4 Inverse of a Function

The ***inverse*** of a function is written as $y = f^{-1}(x)$, and can be found algebraically by switching the x & y variables, and then solving for y → if possible !.

Example 1: p. 49

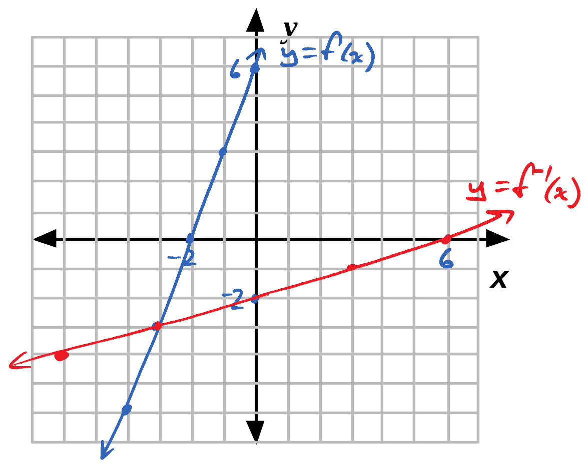
a) find $f^{-1}(x)$ for $f(x) = 3x + 6$, *and graph both on the same set of axes:*

$x = 3y + 6$

$x - 6 = 3y$

$y = \frac{x}{3} - 2$

$\therefore f^{-1}(x) = \frac{x}{3} - 2$



Notice that the x & y intercepts of the function become the y & x intercepts of inverse, and we have switched the domain and range.

b) $f(x) = x^2 - 4$

$x = y^2 \quad 4$

$x + 4 = y^2$

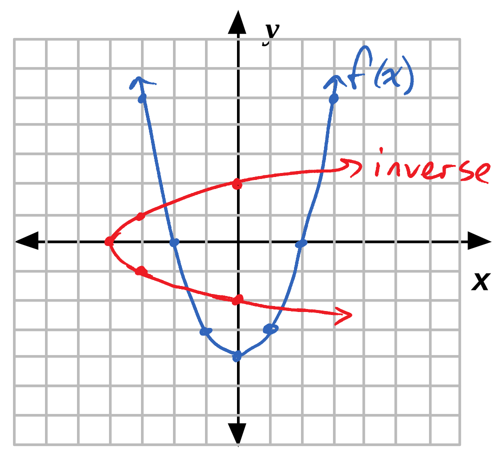
$y = \pm\sqrt{x+4}$

must be ± if you have to introduce the root

∴ this is 2 different functions

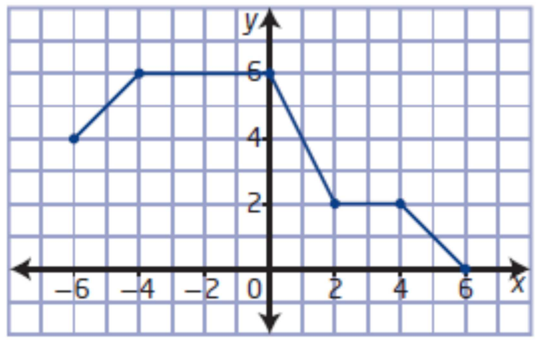
To graph, $Y_1 = \sqrt{(x+4)}$

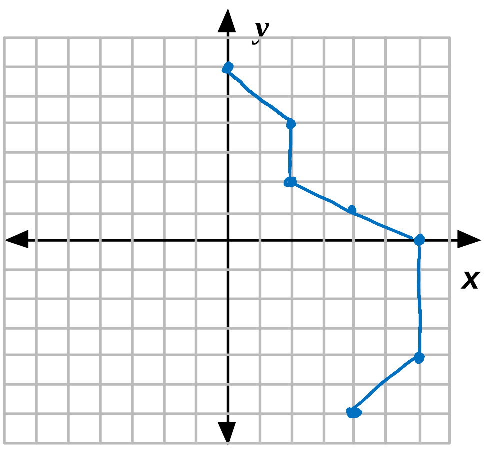
$Y_2 = -\sqrt{(x+4)}$



Graphically, the inverse is a reflection over the line __$y = x$__, which is a diagonal 45^0 line. Because of this, the inverse may not __necessarily be a function__, and sometimes the notation __$x = f(y)$__ is used instead. Invariant points will occur where the graph intersects __the line $y = x$__, and the mapping notation for the inverse is __$(x, y) \rightarrow (y, x)$__.

Example 2: p. 46 – Graph the inverse of:





	function	inverse
Domain:	$[-6, 6]$	$[0, 6]$
Range:	$[0, 6]$	$[-6, 6]$

In this case, the original is a function as it passes the __vertical Line Test__, but the inverse is __not a function__. If the original passes both the vertical & horizontal line tests, then it is called a __one-to-one function__, and we know that the inverse will also be a function.

On the graphing calculator:

Y_1 = function

2[nd] PRGM gives DRAW menu

8:DrawInv

Y1 – this comes from VARS, then right, then enter twice

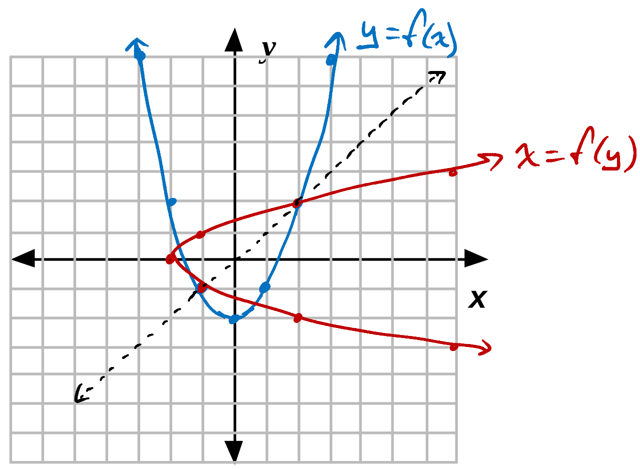
Make sure you finish with ENTER to give the command.

Example 3: p. 48 Consider the function $f(x) = x^2 - 2$.

a) graph the function $y = f(x)$. Is the inverse a function?

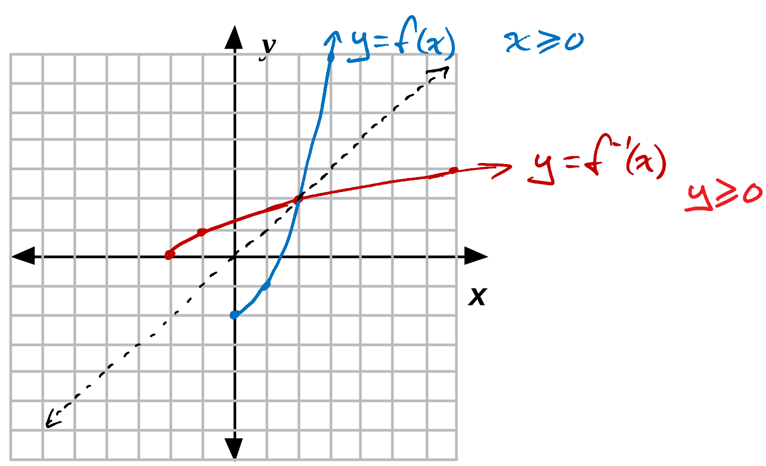
No, as $f(x)$ fails the Horizontal Line Test

b) graph $x = f(y)$ on the same axes.



c) graph the function & inverse again, but introduce a restriction so that both are functions.

just use half of the parabola – $x \geq 0$



Assignment: p.51 # 1, 3-6, 9ace, 12ace, 15, C2