

6.2 Multiplying & Dividing Rational Expressions

Link the Ideas – p. 323 – Multiplying rational numbers

$$\begin{aligned}\left(\frac{5}{8}\right)\left(\frac{4}{15}\right) &= \frac{(5)(4)}{(8)(15)} \\ &= \frac{\cancel{(5)}(\cancel{4})}{(\cancel{4})(2)(\cancel{3})(3)} \\ &= \frac{1}{6}\end{aligned}$$

$$\begin{aligned}\left(\frac{4x^2}{3xy}\right)\left(\frac{y^2}{8x}\right) &= \frac{(4)(x^2)(y^2)}{3(x)(y)(8)(x)} \\ &\quad x \neq 0 \quad y \neq 0 \\ &= \frac{\cancel{4}(x^2)(\cancel{y})(y)}{3(\cancel{x})(\cancel{y})(\cancel{4})(2)} \\ &= \frac{y}{6}\end{aligned}$$

We can perform the same operations with rational expressions if we factor first, then look for common factors to divide out.

Example 1: Multiply and leave in simplest form. Identify non-permissible values.

$$\begin{aligned}&\frac{x^2 - x - 12}{x^2 - 9} \times \frac{x^2 - 4x + 3}{x^2 - 4x} \\ &= \frac{(x-4)(x+3)}{(x+3)(x-3)} \times \frac{(x-3)(x-1)}{x(x-4)} \\ &= \frac{(x-1)\cancel{(x-3)}\cancel{(x+3)}\cancel{(x-4)}}{x(\cancel{(x-3)}\cancel{(x+3)}\cancel{(x-4)})} \\ &= \frac{x-1}{x} \quad \text{or} \quad 1 - \frac{1}{x}\end{aligned}$$

& restrictions $x \neq 0, -3, 4$

⦿⦿ If at any point we are expanding $(x^2 - x - 12) \times (x^2 - 4x + 3)$ we are probably doing the wrong thing!

Factor first

Remember, with dividing fractions, it is easier to multiply by divisor's reciprocal.

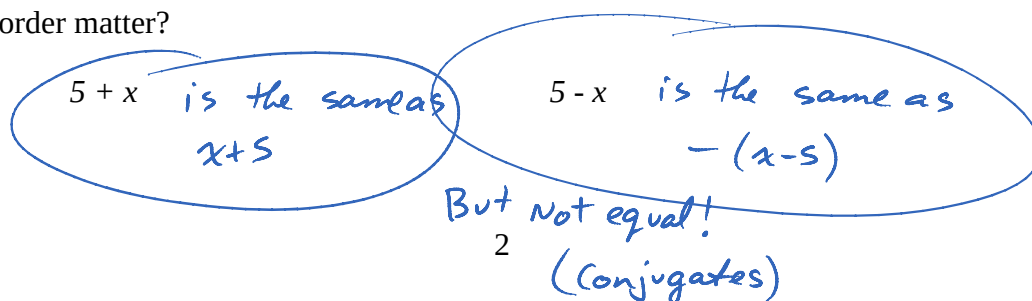
e.g. $\frac{5}{3} \div \frac{1}{6}$ can be changed to $\frac{5}{3} \times \frac{6}{1} \rightarrow \frac{30}{3} = \underline{\underline{10}}$

Example 2: Determine the quotient in simplest form, and give non-permissible values.

$$\begin{aligned} & \frac{x^2 - 4}{x^2 - 4x} \div \frac{x^2 + x - 6}{x^2 + x - 20} \quad \text{factor everything!} \\ &= \frac{(x+2)(x-2)}{x(x-4)} \div \frac{(x+3)(x-2)}{(x+5)(x-4)} \\ &= \frac{(x+2)(\cancel{x-2})}{x(\cancel{x-4})} \times \frac{(x+5)(\cancel{x-4})}{(x+3)(\cancel{x-2})} \\ &= \frac{(x+2)(x+5)}{x(x+3)} \quad x \neq 0, -3, 4, 2, -5 \end{aligned}$$

$$\begin{aligned} & \frac{c^2 - 6c - 7}{c^2 - 49} \div \frac{c^2 + 8c + 7}{7c + c^2} \\ & \frac{(\cancel{c-7})(\cancel{c+1})}{(\cancel{c-7})(c+7)} \times \frac{c(\cancel{7+c})}{(\cancel{c+7})(c+1)} \\ &= \frac{c}{c+7} \quad c \neq \pm 7, -1, 0 \end{aligned}$$

Does the order matter?



Example 3: Simplify and give non-permissible values:

$$\frac{2m^2 - 7m - 15}{2m^2 - 10m} \div \frac{4m^2 - 9}{6} \times (3 - 2m)$$

$$\frac{(2m+3)(m-5)}{2m(m-5)} \times \frac{6}{(2m-3)(2m+3)} \times -1(2m-3)$$

$$= \frac{-6}{2m}$$

$$= \frac{-3}{m} \quad m \neq 0, 5, \pm \frac{3}{2}$$

Key Ideas

- Factor each numerator & denominator, then divide out Common Factors.
- Convert division to multiplication by multiplying by divisor's reciprocal.
- No denominator can equal zero. In $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$ the non-permissible values are

$$B \neq 0, D \neq 0, C \neq 0$$

Assignment: p.327 # 1, 2, 5-8, 15, 16