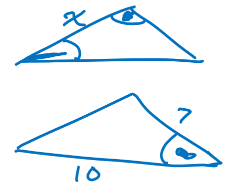
Name___________
Date_____________

# 2.3 The Ambiguous Case

When considering triangles, we can classify them by what information is given:

AAA – all 3 angles
ASA – 2 angles & the included side   e.g.
SAS – 2 sides & the included angle   e.g.
ASS or SSA – 2 sides & an outside angle
← AMBIGUOUS!

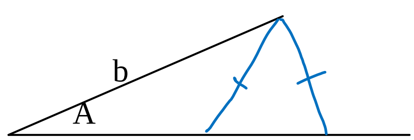



With the ***ambiguous case***, there may be 2 possible solutions.

Using a calculator to find a angle,

sin ϴ =0.5 → ϴ = 30° or ϴ = 150°
sin ϴ =0.707 → ϴ = 45° or ϴ = 135°
sin ϴ =0.866 → ϴ = 60° or ϴ = 120°

What is the relationship? two possible angles add to 180°,
so with the ambiguous case we consider an acute possibility & an obtuse one.
(http://www.mnwest.edu/fileadmin/static/website/dmatthews/Geogebra/AmbiguousCase01.html)

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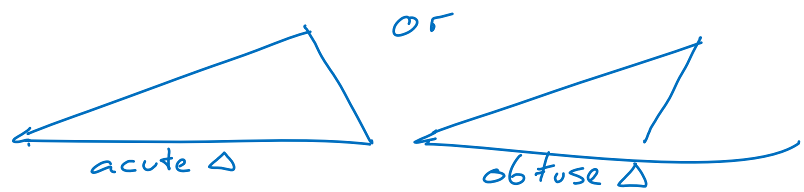



***Example 1:*** Solve ΔABC:

A =37° a = 8 b =12

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{12} = \frac{\sin 37°}{8}$$

$\therefore B = 65°$ or $115°$

Now $\frac{\sin 78}{c} = \frac{\sin 37}{8} \longrightarrow c = 13$

∠A = 37° a = 8
∠B = 65° b = 12
∠C = 78° c = 13

OR

∠A = 37° a = 8
∠B = 115° b = 12
∠C = 28° c = 6.2

Both work

We should always check the answers to ensure the triangle is correct.

i.e. longest side is opposite <u>largest angle</u>

shortest side is opposite <u>smallest angle</u>

***Example 2:*** Solve ΔABC:

A = $82^o$ a = 8.3 c = 7.4

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 82^\circ}{8.3} = \frac{\sin C}{7.4}$$

$C = 62^\circ$ or $\cancel{118^\circ}$

No! Too Big!

$\angle A = 82^\circ$ $a = 8.3$

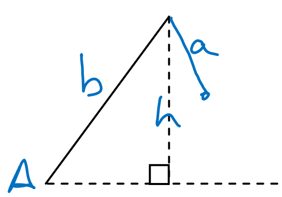
$\angle B = 36^\circ$ $b = 4.9$

$\angle C = 62^\circ$ $c = 7.4$

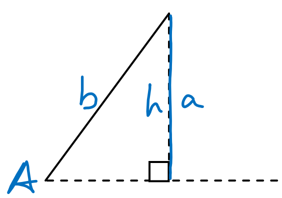
$$\therefore \frac{\sin 82^\circ}{8.3} = \frac{\sin 36^\circ}{b}$$

$b =$

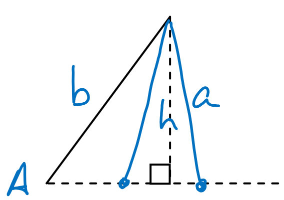
How do we know when we have 2, 1 (<u>or no</u>) solutions? Consider the possibilities for starting with angle *A*, side *b* and side *a*:




No Solution as $a < h$

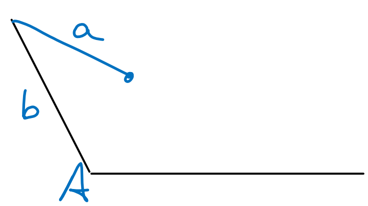



One Solution when $a = h$ (right △)

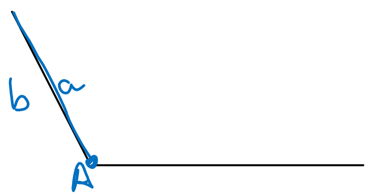



Two Solutions when $h < a < b$

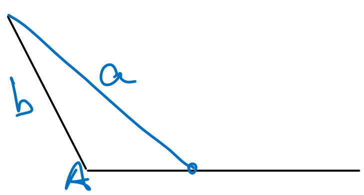
What if the starting angle *A* is obtuse?  Once again, there are 3 cases:




No Solution $a < b$



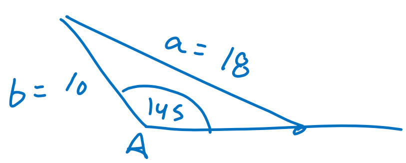

No Solution $a = b$




One Solution $a > b$

***Example 3:*** Determine whether there are 1, 2 or no solutions for the triangle:

$A = 145^o$ $a = 18$ m $b = 10$ m




One Solution

***Assignment:*** p. 108 add on # 4, 6 -10, 24